1. Judging from its title, the system known as First Degree Entailment has as its main target the correct analysis of the entailment relation. Wanting an analysis of entailment in the aftermath of Lewis’ strict implication analysis indicates that the proponents of FDE think that Lewis’ analysis is inadequate. In fact, this is precisely what FDE theorists do think. Their principal objection was that Lewis’ analysis validates ex falso quodlibet. As we have seen, people who want to block the derivation of ex falso must do two things.

- They must change the Lewis definition of entailment.
- They must block Lewis’ proof of ex falso.

To do the latter, they must also find a non-question-begging reason to reject one of the following rules.

- Simplification
- Addition
- Disjunctive Syllogism

As we know, most people who think that ex falso is a mistake gravitate to the view that DS is the defective proof-rule in Lewis’ demonstration.

We can sum this up by saying that a desire for a logic of entailment that would repair the deficiencies of Lewis’ account would have as its main agenda attempts to

- block ex falso
- discredit DS

2. With these objectives in mind, it might strike us as odd that FDE should have been structured in the way that the Routleys and their colleagues have arranged. Whether we opt for the relational, many-valued or possible worlds semantics for FDE, it is hard to see how any of their respective “signatures” delivers the goods in a direct way for FDE’s main project. In particular,
• How does the desire to block \textit{ex falso} and invalidate \textit{DS} motivate \textit{either} a
dialetheic semantics, \textit{or} a many-valued approach \textit{or} an intensional approach
to negation?

It begins to look again as if the two principal objectives of a non-Lewisian analysis
of entailment are achieved in \textit{FDE} as byproducts. In other words, it appears as
if the blocking of \textit{ex falso} and the disabling of \textit{DS} fall onto the category of \textit{objets trouvés} of \textit{FDE}. The question is, \textit{Why}?

3. Before taking this question up, it is historically interesting to note the “four corners”
approach of 6th century Indian logician Sanjaya. Whereas the Western approach
divides sentences into the True and the False, Sanjaya divides them into the True,
the False, Both and Neither. The same is true of \textit{FDE}. Is there anything new
under the sun? (See here, P.J. Raju, “The Principle of Four-Cornered Negation
in Indian Philosophy”, \textit{Review of Metaphysics}, 7 (1954), 694–713; see also Jonaran-
dan Ganeri, “Indian Logic”, in Dov M. Gabbay and John Woods, editors, \textit{Hand-
book of the History of Logic}, vol 1., \textit{Greek, Indian and Arabic Logic}, Amsterdam:

4. The question we’re attempting to answer is, in effect, this:

• Given \textit{FDE}’s motivating targets, isn’t the four-cornered aspect of \textit{FDE} a
kind of over-kill?

To answer this question, let’s look again at Lewis’ proof

1. \(A \land \neg A\) hypothesis
2. \(A\) \hspace{1em} (1), simp.
3. \(A \lor B\) \hspace{1em} (2), add.
4. \(\neg A\) \hspace{1em} (1), simp.
5. \(B\) \hspace{1em} (3),(4) DS

Given that one wants to discredit this proof, consider the following two objections.

\textit{The Cologne Objection}

(a) In line (1) of the Lewis proof, \(A \land \neg A\) is assumed.

(b) In assuming \(A \land \neg A\), one assumes \(A\) and one assumes \(\neg A\). (In other words,
assumption is closed under conjunction.)

(c) In assuming \(A\), one assumes that \(A\) is true. In assuming \(\neg A\), one assumes
that \(\neg A\) is true.

(d) But if \(A\) and \(\neg A\) are both true, the truth of \(\neg A\) can’t exclude the truth of \(A\).

(e) Hence at (3), (4), \(DS\) fails.
**The Routley Objection**

(a) If we look at line (3) of the Lewis proof, we see a disjunction whose disjuncts are wholly irrelevant to one another. The only sense of $\lor$ that allows this kind of disjunction to be derived from one of its own disjuncts is the extensional, truth functional $\lor$.

(b) Had $\lor$ been used in its more natural or intuitive sense in line (3), it wouldn’t have allowed $B$ to be arbitrary. In that case, $\lor$ would be non-extensional, i.e., intensional.

(c) If we now examine line (5) of the proof, we see that the only sense in which $A \lor B, \neg A$ gives $B$ is when $\lor$ is used intensionally.

(d) Therefore, Addition is valid for the extensional but not the intensional $\lor$, and $DS$ is valid for the intensional but not the extensional $\lor$. The proof is defective because it equivocates on $\lor$.

5. **Replies to These Objections**

I With the Cologne Objective the institution of *reductio* proof stands or falls. Consider the following schema

\[
\begin{align*}
\text{1. } A_1 & \text{ assumed for reductio} \\
\vdots & \\
\text{n } A_n & \\
\text{n + 1 } B \land \neg B
\end{align*}
\]

If assumed-truth is closed under deduction, then at $(n + 1)B$ is assumed true and $\neg B$ is assumed true, hence $B$ cannot exclude $\neg B$. But in that case, $B \land \neg B$ isn’t a contradiction, and every *reductio* fails. The key mistake here is that of thinking that if you assume $A \land \neg A$ to be true you assume that its not a contradiction.

II To take the measure of Routley’s objection, we should remind ourselves of one of the first objections to *ex falso*, viz., that $A \land \neg A$ “has nothing to do” with arbitrary $B$. This led critics of *ex falso* to impose a *relevance constraint on entailment*. Relevance, in this instance, would be sharing of wffs. But if we look at Lewis’ proof, it is easy to see that each line after the first, shares a wff with one or more preceding lines. So, for example, $B$ shares content with $(\neg A, A \lor B)$.

What Routley has done in effect has been to impose a *deeper* constraint on disjunction. He thinks that for some uses of $\lor$, $A \lor B$ holds only if there is some relevance between $A$, $B$ (only if, so to speak, they are “genuine alternatives”).

This is fine. But Routley never manages to show why the Lewis proof can’t go through if $\lor$ is allowed to be extensional throughout.

So, if we take a direct, head-on approach to Lewis’ proof, it is hard to discredit it. However, there is no denying it, that if you rig your logic so that $DS$ can’t hold
and *ex falso* can’t be validated, then *in that logic* you’ve got what you wanted. But notice that

- it is one thing to prevent *DS* and *ex falso* from being provable in a system of your own devising.
- it is another thing to have shown that they should not be allowed to hold.

As things now stand, it looks as if the attack on *DS* is entirely *ad hoc*. But is it? In 1959, A.R. Anderson and Nuel D. Belnap, Jr. published “A Simple Treatment of Truth Functions” in the *Journal of Symbolic Logic*. Two things are important about this piece.

- It took it for granted that the connectives of propositional logic were truth functional.
- It noted in a footnote (almost in passing) that *DS* couldn’t be derived in their treatment.

Their *later* judgement that *DS* holds only for the intensional ∨ is likewise significant.

- If ∨ is intensional, the logic in question can’t be truth functional.

This helps explain at least possible worlds semantics for *FDE*. There ¬ is expressly intensional. But if you’ve intensionalized your connectives, *of course*, various of the classical (i.e., extensional) rules will fail.

6. I have been suggesting that *FDE*’s approach to a good or “common sense” theory of entailment is largely tactical. On the face of it, its dialetheism considerably overdetermines its central objectives of refuting *ex falso* and disarming *DS*. True, every dialetheic system is paraconsistent, but assuredly not *vice versa*.

7. Here is Quine on paraconsistent, as well as dialetheic, responses to *ex falso*.

   My view . . . is that neither party knows what he is talking about. They think they are talking about negation, ‘¬’, ‘not’; but surely the notation ceased to be recognizable as negation when they took to regarding some conjunctions of the form ‘p · ¬p’ as true, and stopped regarding such sentences as implying all others. Here, evidently, is the deviant logician’s predicament: when he tries to deny the doctrine he only changes the subject (Quine, *Philosophy of Logic*, 1970, p. 81).

Consider in this regard the possible worlds interpretation of ¬. It is an interpretation that modalizes negation, and it does so with a star semantics that has no recognizable intuitive interpretation. In a sense, then, the price one pays for an *FDE* “paradox”-free account of entailment is that “negation ceases to be recognizable”.
Here is something else to be thinking about. It might have struck us as puzzling that \( FDE \) should have decided to make \( \neg \) an intensional operator. We might think it preferable (“less strategic”) to try to disarm \( DS \) head-on. But consider this:

8. The implication-negation fragment \( R \rightarrow \) of the basic Anderson-Belnap system \( R \). In it \( ex falso \) fails. It also delivers coherent intensional accounts of

- relevant proof
- relevant deducibility
- relevant entailment

It also goes over to a cut-free Gentzen formulation.

Suppose now that we add to \( R \rightarrow \) the extensional connectives \( \wedge \) and \( \vee \). Then in the resulting system, all the above features are lost. This puts us in mind of Quine’s admonition against tinkering with the connectives: “Tamper with one, and you tamper with them all”.

In a way, this vindicates the “strategic” approach of \( FDE \). You can’t succeed against \( DS \) unless you are prepared to give up truth functional logic. So we might as well “tamper” with the connectives right up front.

On the other hand, is it worth the loss of truth functional logic just to spike the cannons of \( ex falso \)?