A Note on Strict Implication

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We said at the beginning that it has been widely-held historically that logic investigates *consequentialist reasoning*. One reasons in this way when one draws consequences from a set of priors. This imposes two tasks on logicians.

1. They must attempt to determine what the consequences of a set of priors are.
2. They must attempt what consequences of a set of priors ought to be drawn.

As we saw, the ideal model theory says that, ideally, all consequences should be drawn. The proper subset model says that only a proper subset of consequences should be drawn. C.I. Lewis never spoke of this issue in these terms, but it can reasonably be supposed that he himself would favour the proper subset model. For Lewis, consequence is strict consequence, i.e., the converse of strict implication. So whenever \( A \) strictly implies \( B \), \( B \) is a strict consequence of \( A \). The so-called paradoxes of strict implication bear on this interpretation of Lewis’ position. Lewis was adamant that any \( B \) whatever was indeed a strict consequence of any contradiction whatever. But he certainly did not think that every proposition whatever should be believed on the basis of an inconsistency that might afflict one’s priors. In fact, we may take it that Lewis’ view was that nothing whatever should be believed on the basis of a contradiction. In other words

1. Although \( A \land \neg A \) has arbitrarily many strict consequences, none of them should be drawn on the basis that \( A \land \neg A \).
2. Since, in this case, zero consequences should be drawn, the set of consequences that should be drawn is the null set. Since the null set is a proper subset of any non-empty set, the set of consequences that should be drawn is a proper subset of the set of consequences that \( A \land \neg A \) possesses.

Critics of strict implication lodge one or other of two objections. These criticisms are probably related to one another.

*Objection 1*  No proposition properly implies another if they are wholly unrelated to one another. Since it is provable in the Lewis systems that
irrelevant propositions may strictly imply one another, strict implication can’t be proper implication (or, as we shall now say, entailment).

**Objection 2** Since no one in his right mind would draw an arbitrary consequence from a contradiction, no such proposition can be a consequence of a contradiction.

**Comment**

- *Concerning (1)* If we pressed the critic as to why relevance should be a condition on entailment, he would tell us that competent reasoners don’t infer irrelevancies.

- *Concerning (2)* This clearly suggests that the critic holds with the ideal reasoner model. The same would appear to be the case with regard to the reason for advancing objection (1).

This suggests a third option. We might call it reconciliation. It may be expressed as follows.

1. When Lewis claims that *ex falso* is true there is something that he is clearly right about.

2. When Lewis’ critics claim that *ex falso* is false, there is something that they are clearly right about.

Lewis is right about entailment. His critics are right about inference. Accordingly, the proper subset model of consequentialist reasoning is correct.