1. Objections

(1) A system of modal propositional logic is a *bona fide* logic only if it has a quantificational extension.

(2) S5, for example, has a quantificational extension, but there is something disastrously wrong with quantified modal logic (*QML*).

(3) The difficulties of *QML* are of such a character as to show — or strongly indicate — that they derive from the modal peculiarities of the embedded propositional system.

(4) So not only is *QML* a failure, so too is any propositional system which extends its hospitality to the modalities.

Quine forwards six arguments against *QML*.

   The statement (1) (\(\exists x\))N(x>7) is curious. "... would 9, that is, the number of planets be necessarily greater than 7?"


   If we accept the substitutional account of the quantifiers then we have a problem. Let c be the congruence relation on morning star (MS), evening star (ES), and Venus, such that

   (1) Ms\(c\) ES\(v\) N(MS\(c\) MS)
   (2) Esc\(c\) ES\(v\) 5N(ES\(c\) MS)

   Thus we get

   (3) (\(\exists x\)) (xeES \(v\) N(xeMS) from 1)
   (4) (\(\exists x\)) (xeES \(v\) 5N(xeMS) from 2)

   But right hand 3 and right hand 4 are one another's contraries and cannot be satisfied by the same object, Venus. In fact there must be at least three different objects occurring in our congruence relation. Astronomy is over-turned.

   One and the same number \(x\) is uniquely determined by the condition
   (1) \(x=\ / x + / x + / x \ne / x\)
   and
   (2) There are exactly \(x\) planets.
   But (1) has 'x>7' is a necessary consequence and (2) does not.
Suppose those we try to get around this difficulty as follows. We restrict our universe of discourse to objects $x$ such that any two conditions uniquely determining $x$ are analytically equivalent, i.e., we put it that

\[(i) \text{ } (\forall y)(Fy/ y = x) \equiv N(\forall y)(Fy/ y = x)\]

But introducing ‘$\alpha = z$’, for ‘$F\alpha$’ in (i) and then simplifying and closing, we get

\[(\forall x)(\forall y) (x = z) \equiv N(x = z).\]

Quine gives an argument to the same effect. He also claims that essentialism, which is wholly untenable, is unavoidable in any system of $\text{QML}$. For if we have

\[(1) \text{ } NFx \lor Gx \lor NGx\]

then we have

\[(2) \text{ } (\exists x)(NFx \lor Gx \lor Gx)\]

putting ‘$x = y$’ for ‘$Fx$’ and ‘$x = x.p$’ for ‘$Gx$,’ where $p$ is any contingent truth.

If we accept

\[(1) \text{ } (\forall y)(Fy/ y = x) \lor (\forall y)(Gy/ y = x) \equiv N(\forall y)(Fy/ Gy)\]

and if

\[(2) \text{ } p\]

is any true sentence, and

\[(3) \text{ } w=x\]

then we have

\[(4) \text{ } (\exists y)(p \lor y = w/ y = x)\]

and

\[(5) \text{ } (\exists y)(y = w/ y = x)\]

Then, putting ‘$p.\tilde{\alpha} = w$’ for ‘$F\tilde{\alpha}$’ and ‘$\tilde{\alpha} = w$’ for ‘$G\tilde{\alpha}$’ in the first formula above we get

\[(6) \text{ } (\exists y)(p \lor y = w/ y = x) \lor (\exists y)(y = w/ y = x) \equiv N(\exists y)(p \lor y = w/ y = w)\]

But (6), (5), (4) jointly imply

\[(7) \text{ } N(\exists y)(p \lor y = w/ y = w)\]

and (7) implies

\[(8) \text{ } p \lor w = w/ w = w\]

which implies

\[(9) \text{ } p.\]

But since (7) arises from a necessary truth, $p$ must be necessary. Hence

\[(10) \text{ } Np.\]

Modal distinctions collapse.
Quine’s objections to QML all turn on one or more of the following matters: (a) essentialism, (b) definite descriptions, and (c) the identity relation. If we are properly to take the measure of his complaints, it is important to be clear about these things.

**ESSENTIALISM:** Quine regards the doctrine as unintelligible. He says that there is no defensible reason to say of a bicycling mathematician that he is necessarily rational and only contingently two-legged.

Virtually all essentialists have wanted to mark the following distinction:

(i) Necessarily Socrates is rational \([N(Fa)]\)

and

(ii) Socrates is necessarily rational \([F^N a]\)

In (i) we have a *de dicto* modality, in which necessity is represented as a trait of sentences, in such a way that (i) is true if and only if the sentence “Socrates is rational” is true in all possible worlds. On the other hand, (ii) imputes a *de re* modality, in which the property of being necessarily-rational is attributed to the object Socrates, true of him in every possible world in which Socrates exists.

The principal difference between (i) and (ii) is that (i) imputes to Socrates necessary existence, whereas (ii) does not. On its most natural reading, (ii) says not that Socrates exists in every possible world, but rather that there is no possible world in which he exists but fails to be rational.

Accordingly, we may propose a definition of essentialism by way of necessity *de re*:

**Def:** a is F essentially iff a \(F^N a\) iff \((a \text{ is F}) \land N(5(a \text{ is F})e5\Rightarrow x(a=x))\).

Thus, any property a thing possesses essentially is one whose loss would extinguish that thing's self-identity.

**DEFINITE DESCRIPTIONS:** Definite descriptions are expressions in the form “the so-and-so” (e.g., “the present Queen of England,” the “only even prime,” the “sum of 2 and 7,” the “the wife of Jean Chrétien,” etc.) Definite descriptions form a subset of singular terms. They are noun phrases in referring position in singular sentences, or in embedded singular clauses. As such, definite descriptions are subject to what might be called “the standard policy” on singular referring terms.

1. **The Substitutivity of Identicals:** Let us say that if \(\alpha\) and \(\beta\) are terms for which \(\alpha = \beta^k\) is true, then \(\alpha\) and \(\beta\) are *co-referential* terms. Then for any pair of co-referential terms \(v_1\) and \(v_2\), \(v_1\) may be substituted for any occurrence of \(v_2\) in any extensional sentential context, *salve veritate* (i.e., in a truth-preserving way).

2. **The Principle of Semantic Constancy:** If a sentence \(\psi\) arises from a sentence \(\Phi\) by substitution at one or more places of co-referential terms \(\alpha\) and \(\beta\), then where \(\psi\) differs from \(\Phi\) (vis., at \(\alpha\) and \(\beta\)), \(\alpha\) and \(\beta\) may differ from one another only syntactically; that is, both \(\alpha\) and \(\beta\) must perform the *same semantic function* in both \(\Phi\) and \(\psi\).

Consider the terms ‘Cicero’ and ‘Tully’ made co-referential by virtue of the truth of

\[(1) \quad \text{Cicero}=\text{Tully}\]
It is known that

(2) Cicero was a Roman orator.

Then, by the Substitutivity of Identicals, we may infer

(3) Tully was a Roman orator

but only if the semantic role of ‘Cicero’ in (2) and of ‘Tully’ in (3) is precisely the same (for otherwise, (1) could not be true). In particular, Semantic Constancy precludes that in (2) ‘Cicero' refers to a city in Illinois, near Chicago; and it likewise precludes ‘Tully’ in (3) referring to a recent Dean of Arts at the University of British Columbia.

IDENTITY: Essentialists and non-essentialists alike want to recognize a distinction exemplified by

(1*) Cicero=Cicero

and

(1) Cicero=Tully

On the face of it, this is a problematic distinction. For one thing,

(1**) \(\forall x(x=x)\)

can be taken as a necessary truth de dicto (for even in the empty world, (1**) is a vacuously true; that is, it will still be true that \(\exists y(y\neq y)\)). And although (1*) looks like a straightforward universal instantiation of (1**) it cannot be both true and necessary de dicto, since it is not a necessary truth that Cicero exists. Therefore, we say that (1*) is necessary de re. On the other hand, (1) is purely contingent. It is necessary neither in the de dicto sense nor the de re sense. Consider now the following claim:

(4) The only way in which it could be false that Cicero is identical to Cicero is if Cicero did not exist.

If we apply to (4) the Substitutivity of Identicals, and if Semantic Constancy is also honoured, we obtain

(5) The only way in which it could be false that Cicero is identical to Tully is if Cicero did not exist.

But this is wrong. Or anyhow on its most natural interpretation it is wrong. For here we want to say that one way in which it could be false that Cicero is identical to Tully is if the existent Tully were named ‘Charlie' rather than ‘Cicero'.
Hence if (5) is false, Semantic Constancy is breached. So there must be a pair of contexts in which it is clear that the semantic roles of ‘Cicero’ and ‘Tully’ differ from context to context. Two such contexts are (1*) and (1). For if Semantic Constancy is honoured at (1*) and (1), then both are necessary de re or both are purely contingent. But this is not what they are on their most natural readings. Hence ‘=‘ does not occur in (1) as the sign for numerical identity. That is, (1) is equivalent to

(5) \(\exists x (\text{Cicero names } x \vee \text{Tully names } x)\).

When a sentence such as (1) stands so to a sentence such as (5), logicians say that there is a term in (1) which is contextually eliminated in this process. Russell called an expression which is contextually eliminable from a context an incomplete symbol in that context. In the present example, we see that ‘=‘ is an incomplete symbol in (1) but not in (1*). Hence if we say that (1*) is a statement of numerical identity, (1) is not a statement of numerical identity.

We now have the wherewithal to appraise Quine’s six objections to QML.

2. Replies

Objection One [Quine, 1953a]
Consider:

(1) \(\exists x N(x > 7)\).

(2) \(N(9 > 7)\)

and

(3) \(N (\text{The number of planets} > 7)\)

There is no particular reason to give ‘N’ in (1) a reading de dicto. It is already problematic whether even (2) is true (where ‘N’ is de dicto), short of ascribing necessary existence to the integers. So the commonplace claim that there are some things that are necessarily greater than 7, should be analyzed as

(1*) \(\exists x (x > N\ 7)\).

As for (3), if the Principle of Semantic Constancy is honoured in the move from (1) to (3), then it is essential that in (3) the definite description ‘the number of planets’ names the number 9. This is because (3) comes from (2) by the Substitutivity of Identicals on

(4) The number of planets = nine.

If ‘=‘ is the identity relation in (4), and if (4) is true, it must be flanked by co-referential terms. Hence if (2) were all right, so too would (3) be. But (3) does not look right. On its most natural reading it is synonymous with
Necessarily, there are more than seven planets.

Now (5) is wrong for both de dicto and de re reading of ‘N.’ But it does not matter. We cannot get (5) from (1) except by breaching Semantic Constancy.

**Objection Two** [Quine, 1943, p. 123]
It is essentially the same objection as before. I give it essentially the same reply.

**Objection Three** [Quine, 1947, p. 47]
Let $c$ be the congruence relation on the set $\{\text{The Morning Star, The Evening Star, Venus}\}$. Then we have both

(6) $(\text{MscES})vN(\text{MSeMS})$

and

(7) $(\text{EscES})v5N(\text{ESeMS})$.

But from (6) we have

(8) $\triangleright x(\text{xESE})vN(\text{xEMS})$

and from (7) it follows that

(9) $\triangleright x(\text{xESE})v5N(\text{xEMS})$.

If we want to say that the Morning Star is necessarily self-congruent, the necessity must be read as de re (otherwise the MS would exist necessarily). Proposition (6) is a completely unacceptable reading of this innocent claim. Moreover, if in (7) Semantic Constancy is honoured, then its rightmost clause is as acceptable or not as the rightmost clause of (6). Of course

(10) MS=ES.

But if this is to be a purely contingent claim, it is equivalent to

(11) $\triangleright x(\text{\textquoteleft MS' names } x\text{'ES' names } x)$.

As we see, we get (8) and (9) only if we violate Semantic Constancy, which we must not do if (6) is a genuine statement of numerical identity. Of course, on its most natural reading it is not. That is, ‘=’ is contextually eliminable in (10) in favour of (11). In which case “MS $c$ ES” is not even necessary de re.

**Objection Four** [Quine, 1953a, p. 149]

One and the same object is uniquely determined by the conditions
(12) $x = / x + / x \neq / x$

and

(13) There are exactly $x$ planets.

But (12) has a necessary consequence.

(14) $x > 7$

whereas (13) does not.

What Quine is trying to say is that 9, which satisfies (12), is necessarily greater than 7; and that 9, which satisfies (13), is not necessarily greater than 7. Quine is confused. Being necessarily (de re) greater than 7 is a trait of 9 each time. What we do not have is

(15) There are necessarily more than seven planets

i.e.,

(16) There are of necessity more planets than seven.

Now (16) is certainly not true. On the other hand, (16) is not derivable from anything that goes before.

Suppose we grant that

(17) $(\forall y)(Fy / y = x \land (\forall y)(Gy / y = x) \land (\forall y)(Fy / Gy))$

Then we obtain

(18) $(\forall y)(Fy / y = x) \land (\forall y)(Fy / y = x)$

which gives

(19) $(\forall x)(\forall y)(x = z \land (\forall x)(y = z))$.

The conclusion that there are no contingent truths of identity is true, not false if Semantic Constancy is honoured and if necessity is construed as de re. For consider

(20) Cicero = $^N$Cicero

which is true, and

(21) Cicero = Tully
which is also true. Then by the Principle of Substitutivity of Identicals, together with Semantic Constancy, we substitute ‘Tully,’ for the second occurrence of ‘Cicero’ in (20) and obtain

\[(22) \text{Cicero} =^N \text{Tully}\]

which is true. Of course it is not true on the most natural reading of (21), viz.,

\[(23) \exists x (\text{‘Cicero names } x \vee \text{‘Tully names } x).\]

For on that reading, (22) would be construed as

\[(24) \exists x (\text{‘Cicero names}^N x \vee \text{‘Tully names}^N x)\]

which is indeed absurd. But note that (24) does not come from (20) by Substitution. This is because (21) is not a statement of numerical identity. The sign ‘=’ is contextually eliminable in (21), in the manner of (23).

There is a final thing to say against this objection. We saw that (17) was ventured only for domains in which it might plausibly be taken for true. One such domain is the set of natural numbers. But according to the received wisdom, the natural numbers are distinctive: No theorem is contingent. No non-Leibnizean modalist will allow that every true identity whatever is a necessary truth, but this is precisely what he will allow — indeed insist upon — in the domain of natural numbers. Assumption (17) is plausible precisely for those domains in which modal collapse is already a desired consequence. The derivation of (19) therefore fails to stick the modalist with anything he would regard as absurd.

Objection Six [Quine, 1960, pp. 197-198]
Here, too, we begin with assertion (17). Quine goes on to show that modal distinctions collapse under that assumption, i.e., that for all \(\Phi\)

\[(25) \Phi \text{ iff } N \Phi\]

The same point applies to (17). The objection produces no absurdity, but rather the wholly welcome consequence that, e.g., arithmetic truths are necessary. Apart from that, at one place in Quine’s proof, it is asserted that

\[(26) (\exists y) (p \forall y = w / y = x) \vee (\exists y) (y = w / y = x) \vee N (\exists y p \forall y = x / y = w).\]

Note that ‘N’ is in de dicto position. It has no right to be. It belongs in de re position only. But thus positioned, the proof collapses; we cannot validly derive (25). It is well to remember that Quine’s acceptance of (17) is both tactical and highly restricted. It is intended for domains such as the integers, in which unique specification is necessarily unique specification. In such a domain that the number 4 is uniquely determined by the condition “2+2” and by the condition “3+1” gives us all we need to judge the hard equivalence of “is the sum of 2 and 2” and “is the sum of 3 and 1.” But, again, there is nothing whatever surprising that in the theory of such a domain, viz., arithmetic, modal distinctions collapse. That is what every modalist already thinks, i.e., that arithmetic truths are not contingent. This is an ironic twist for Quine; his embrace of an
assumption which he does not like serves to deliver a consequence to which the modalist is already pledged, and in which to sees no reason for concern.

All six of Quine's objections turn on wildly implausible readings of commonplace truths. These objections are so obviously defective and their shortcomings so apparent that it can only be wondered whether Quine has not been trifling with us.