# Methodological Challenges of Instabilities and Complexity

On the methodological discussion of mathematical models in nonlinear sciences and complexity theory

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Today, methodological and epistemological discussions about models and modeling take place within present-day sciences, such as Complex Systems Theory, Nonlinear Dynamics, Chaos Theory, and Synergetics. In this interdisciplinary field an explicit debate about *apriori* conditions for "adequate" or "relevant" models has emerged and is still ongoing. The thesis of my contribution is that the main reason for the methodological reflection on modeling issues (and on the validation, evidence, and explanatory power of models) in the branch of complexity is the broad acknowledgement and the further discovery of instabilities in objects and models.

Today, however, *on the one hand* dynamical and structural instabilities are regarded as the sources for complexity, self-organization and pattern formation. *On the other hand*—and this is the focus of my contribution—instabilities are challenging traditional scientific methodology, in particular one of its most implicit *apriori* conditions: stability. Stability was taken for granted—as *the* apriori condition to qualify a mathematical model as "physically relevant" or as an "adequate" model; stability guarantees methods of approximation and dealing with empirical uncertainties. Today, *ex post* we can identify a "dogma of stability" that has restricted the choice of possible models throughout the history of natural sciences (Guckenheimer/Holmes 1983). However, since the 1960s, instabilities have no longer been disregarded and neglected. In fact, instabilities have turned out to be fundamental in nature, technology, and even in social processes. They are essential for complexity, self-organization, and pattern formation. Therefore, "we shall question the conventional wisdom", Guckenheimer and Holmes state, "that stability is an essential property for models of physical systems."

Nonlinear sciences and complexity theory have built up a methodological framework that enables science to address dynamically and structurally instable objects—this will be shown and reflected in my contribution. Although model construction generally traces back at least to the origin of modern sciences, today's mathematical modeling, computer simulations, and numerical experiments are a relatively new practice of knowledge production that emerged in the 1960s. This knowledge production, although intertwined with the development of computer technology, is mainly induced by the acknowledgement of instabilities and by the need to methodologically account for them. In this field scientists regard their approach more as a "model-building" than as "theory"- or "law"- centered. This finding will be combined with some prominent illustrations that philosophers of sciences have brought up: e.g., Cartwright's 1.000-Dollar-bill, however she does not realize the nomological core of this phenomena, namely instabilities. Considering these examples I will argue from a systematic point of view for a position of a "contextual structural realism". – The outline of my contribution is as follows:

(1) *Instabilities:* I will show that nonlinear systems (e.g., objects as well as models), even those with just a few degrees of freedom, can exhibit mainly two types of instability: dynamical and structural instability. *Dynamical instability* means that the dynamics of a model or a physical object is time-continuously sensitive (e.g., chaotic attractor). *Structural instability* does not refer to the initial points, but to the structure of the objects or the models itself: if one changes a model slightly or disturbs the object infinitesimally, the overall dynamics changes qualitatively—the equivalence class of models is not preserved. In addition, I will show how these two kinds of instabilities can be related, although there does not exist a unified mathematical theory of instabilities of the very different processes in various fields. To elaborate this, examples will be provided and some elements of the historical discussion in mathematics (differential topology, dynamical systems theory) will be highlighted showing the problems of an *instability-unification* (global and local stabilities; landscape stabilities; stabilities with regard to spatio-temporal properties; stabilities in stochastic processes; instabilities in thermodynamics as a source for irreversibility – and in general: bifurcation theory). I will end this subsection by pointing out a plurality of definitions of "instabilities". An interesting analogy to the plurality and contextuality of the term "chaos" can be shown (cp. Brown/Chua 1996/1998).

(2) *Methodological challenges:* The focus on stability has been complemented by, and supported by, a theoretical focus on the role of symmetry, which has been very successful in the investigation of fundamental universal laws of dynamics. So that, taken together, the symmetry/stability approach has dominated, not least because it also ensures the validity of quantitative methods of approximation (small perturbation theory) and through that dealing with empirical uncertainties. However, *on the one hand* instability also provides the condition for the appearance of symmetry breaking, self-organization and formation of complexity. *On the other hand* instabilities have consequences for understanding the traditional methodological core of sciences. I will explicitly focus on four challenges and treat these issues in an analytical framework: (a) reproducibility / experimentability, (b) predictability, (c) (reductive) explanatory power / economical describability and (d) testability / confirmability. For any instable model that refers to an instabile object's behavior, "details of the dynamics, which do not persist in perturbations, may not correspond to testable [...] properties." (Guckenheimer/Holmes) The "Devil is in the detail", as Batterman has stressed; "approximate reasoning" is limited. The acknowledgement of this methodological challenge is not novel. E.g., Maxwell already pointed out in the 1870s, "in

so far as the weather may be due to an unlimited assemblage of local instabilities, it may not be amenable to a finite scheme of law at all." At Maxwell's time, just a few scientists were aware of this challenge. Today, the problem of and the threat to (HO-) reductionism is broadly acknowledged and deeply discussed within Nonlinear Sciences.

However, there have been different historical epochs trying to deal with these challenges (sect. 3 and 4). – (3) *Neglect and disregard*: Traditionally, throughout the history of science until the late 19$^{th}$ century, the problem was "solved" by—or, to be more precise, it was not perceived as a pressing issue because of—an implicit "Dogma of Stability" (Guckenheimer/Holmes 1983): Thus, no methodological challenge emerged. Instabilities were disregarded and devaluated, although since the Navier-Stokes equation of hydrodynamics, scientists have been, at least to some degree, aware of instable processes in objects and models. In Poincaré's work on the dynamics of the solar system, in Einstein's field equations and, even earlier, in Newton's theory of the moon, Boltzmann's statistic-mechanical formulation of irreversibility, and Maxwell's discussion of stability in his work "matter and motion", the problem of instability was identified. However, even in the 20$^{th}$ century, Duhem and Andronov believed that sciences were threatened by instabilities and thus, models should be restricted to stability: if stability cannot be taken for granted, then it has to be imposed in order to cope with empirical uncertainties and experimental errors. The "stability dogma" was imposed as a strong methodological norm. But this dogmatic requirement served also as a selection criteria for "physically relevant" objects.

(4) *From Stable Theories to Prevalent Properties of Models of Complex Systems*: Today, the threat for a proper methodology is partly reduced and some major problems are resolved. It will shown that present-day methodological discussions deal with *structural*, *generic* and *prevalent* aspects of instable models. The stability dogma is substituted by weaker apriori conditions on models that refer to those physical properties which are only relevant for the specific context in question—a contextualism referring to a "dappled world" emerges ("contextual structural realism", as I will show). "The definition of physical relevance will clearly depend upon the specific problem. This is quite different from the original statement that the only good models are ones with all of their properties preserved by perturbations." (Guckenheimer/Holmes 1983) The weaker conditions are based mainly on *qualitative* ("topological") instead of mere *quantitative* properties or characteristics; such qualitative approaches are based on recently developed and advanced methods in mathematics (nonlinear data analysis, phase space / attractor reconstruction, ... ). Generally, the *qualitative* is highly acknowledged in complexity theory and nonlinear dynamics today. For instance, Mandelbrot calls his "Fractal Geometry" a "qualitative theory of different kinds of similarities" and Thom stresses that his "Catastrophe Theory" is a "qualitative science of Morphogenesis". This traces back to the "qualitative theory of differential equations" (Birkhoff, Poincaré). The qualitative will be critically reflected and discussed with regard to issues such as representation, evidence, confirmation, and explanatory power. A specific type of explanation, the *qualitative explanation* within *contextual boundaries*, emerges—and this will be con-

trasted with the concept of *asymptotic reasoning* (cp. Batterman). For my argumentation I will provide some easily accessible examples, e.g. impact- and stick-slip- oscillators.

Thus, instabilities are to be regarded as one of the most striking nomological focuses in recent methodological discussions about models of complex systems.