

Dialectical Considerations on the Logic of Contradiction: Part I

John Woods
Department of Philosophy
University of British Columbia
1866 Main Mall
Vancouver B.C.
V6T1Z1
e-mail: jhwoods@interchange.ubc.ca

Department of Computer Science
King's College
Strand
London
WC2R 2LS
UK
e-mail: woods@dcsc.kcl.ac.uk

Dialectical Considerations on the Logic of Contradiction: Part I

“Who gathers knowledge/gathers pain”.

Ecclesiastes 1.18

“Do what you will, this world is a fiction
And is made up of contradiction”.

William Blake

“Conversely, by the same token, no statement is immune to revision. Revision even of the logical law of the excluded middle has been proposed. . . .”

W.V. Quine

Abstract

This is an examination of the dialectical structure of deep disagreements about matters not open to empirical check. A dramatic case in point is the Law of Non-Contradiction (LNC). Dialetheists are notoriously of the view that, in some few cases, LNC has a true negation. The traditional position on LNC is that it is non-negotiable. The standard reason for thinking it non-negotiable is that, being a first principle, there is nothing to negotiate. One of my purposes is to show that the first-principle defence of LNC is inadequate. A second purpose is to argue that it flows from this inadequacy that LNC stands or falls on economic considerations, much in the spirit of Quine’s pragmatism about logic generally. This is a tactical victory for dialetheists. It gives them room to make the case against LNC on cost-benefit grounds. As things presently stand, no such case can be considered decisive. But, given that costs and benefits shift with changing circumstances, it is possible that a winning case for the dialetheist may present itself in the future. Notwithstanding the rivalry between consistentists and dialetheists, they share a common opponent. This is trivialism, the doctrine that everything whatever is true. It is an ironic alliance, in as much as the dialetheist’s case against the consistentist can be adapted to a defence of trivialism. How damaging this turns out to be depends on the adequacy of the reasons for the dialetheist’s rejection of trivialism. My further purpose is to show that the damage is slighter than dialetheists commonly believe.

1. *Inconsistency*

Trivialism is the doctrine that everything is both true and false. Near-trivialism is the doctrine that nearly everything is both true and false.¹ Dialetheism is the doctrine that hardly anything is both true and false. Consistentism is the doctrine that nothing whatever is both true and false.² Here “true and false” means “true and false in all the same respects”. In alternative versions, “true and false” is replaced by “true and not true”.

Trivialism would be a correct view if the proof the Curry Paradox were sound.³ Dialetheism would be a correct view if the proof of the Liar Paradox were sound⁴ and the proof of the Curry (or anything else to the same effect) were unsound. Consistentism would be true if no paradoxical proof (or anything else to the same effect) were sound.

Consistentism has been decidedly the dominant position in logic. At its most cavalier, it is a perspective from which the very idea of true contradictions is met with derision and outright dismissal. By classical lights, a true contradiction would necessitate the truth of everything; so, if dialetheism were true, trivialism would also be true.

In the past four decades or so, classical logic has had to contend with a rich and rivalrous pluralism, in some of whose precincts the classical line on inconsistency has been abandoned.⁵ This has cost consistentism not its dominance but rather its cavalierness. What makes this so is that logics that take a nonclassical approach to inconsistency admit of precise articulation in formalisms that are sound and complete, and whose results have found their way into the mainstream journals and monographs published by some of the leading university presses.⁶

Recent developments have indeed put pressure on logicians to reconceptualize inconsistency. One is the growing recognition by belief dynamics theorists, AI researchers and cognitive psychologists of the extent to which competent inferences are drawn from inconsistent databases by actual reasoners in real-life situations.⁷

¹ So-called by Graham Priest in “Could Everything Be True?”, unpublished ms.

² Some of these characterizations are, of course, cardinally inexact. Most dialetheists hold that, although there are arbitrarily many instances of them, there are only a few structures or sources of true contradiction. The Liar proof is a case in point. It proceeds from statements in the form, “This statement is untrue”, with which arbitrarily many others are equivalent: “This statement is untrue”, is true’, “This statement is untrue” is true’, is true’ and so on. Near-trivialism is usually grounded in vague predicates, of which every language has a large but finite number. The sense in which near-trivialism countenances more true contradictions than dialetheism is that near-trivialism acknowledges a much greater number of *sources* of such truths. It should also be noted that the near-trivialism that vagueness purportedly gives rise to is conditional upon there being change in the world.

³ Haskell B. Curry, “The Paradox of Kleene and Rosser”, *Transactions of the American Mathematical Society*, 50 (1941), 454–516; and Haskell B. Curry “The Inconsistency of Certain Formal Logics”, *Journal of Symbolic Logic*, 7 (1942), 115–117. See also George Boolos and Richard Jeffrey, *Computability and Logic*, 3rd ed., Cambridge: Cambridge University Press 1989.

⁴ Alfred Tarski, “The Concept of Truth in Formalized Languages”, in Alfred Tarski, *Logic Semantics. Metamathematics*, J.H. Woodger trans., Oxford: Oxford University Press: 1956; 2nd ed., John Corcoran (editor.), Indianapolis: Hackett 1983; 152–278. Originally published in Polish 1936.

⁵ Not to say that classical logic had an entirely free ride beforehand. Intuitionist and modal logics were stirring significantly well before the 1920s.

⁶ See, for example, Graham Priest, Richard Routley and Jean Norman, editors, *Paraconsistent Logic: Essays on the Inconsistent*, Munich: Philosophia Verlag 1989.

⁷ See Dov M. Gabbay and Anthony Hunter, “Making Inconsistency Respectable, Part I,” in P. Jourand and J. Kelemen, editors, *Fundamentals of Artificial Intelligence Research*, Berlin: Springer-Verlag, vol. 535

Accordingly, the root claim of *paraconsistent logic* is that negation-inconsistency does not imply absolute inconsistency; that is to say, that local inconsistency does not trigger global inconsistency. Paraconsistent logicians are for the most part consistentists. While they insist that a given true contradiction would not entail universal inconsistency, they don't think that any given contradiction actually is true.

The founding insight of paraconsistent logic is nicely illustrated by the fateful discovery communicated by Russell to Frege in 1902. The axioms of intuitive set theory were inconsistent. Every sentence follows classically from those axioms. Surely this is a disaster. It is, and is not. It is a disaster (anyhow a setback) for anyone who seeks a consistent axiomatization of sets. But it would be a catastrophe for set theory only if set theory were blown apart by it. If this were to be so, it would have been so before the Russell set was discovered. That set theory didn't in fact detonate⁸ tells us something important. The presence of a contradiction in intuitive set theory was no bar to the generation of its myriad rich results then and now. So it cannot be true that an inconsistent axiomatization of a discipline shuts the discipline down.⁹ There is another way of saying this. The Russell paradox did not show that mathematicians didn't (indeed couldn't) know how to do set theory. It didn't show that mathematicians knew nothing about sets or about the transfinite. What it showed was they hadn't yet found consistent axioms for those sets.

Most systems of paraconsistent logic are "Hippocratean". In reasoning from an inconsistent set of priors, the reasoner is required to honour the principle *primum non nocere*, that is, do no harm.¹⁰ The conclusion of the reasoning should reflect a state of affairs "no worse" than the state of affairs purported by the inconsistent set of priors.¹¹ What is so striking about the state of pre-paradox set theory is that even in the presence of the contradiction demonstrated by Russell, it managed to do its quite considerable business in full conformity with the Hippocratean imperative.

The catastrophe theory of inconsistency pivots on the metaphors of *blowing apart*, *detonation* and *shutting down*. It is a figuratively powerful trio, all coming to the same thing. Staying with sets as our example, the principal epistemic goal of set theory is to disclose those sentences about sets that are true as opposed to false. When this goal is attained, the theorist has achieved *a knowledge of sets*. This knowledge is

(1991) and Dov M. Gabbay and Anthony Hunter, "Making Inconsistency Respectable, Part II", in M. Clarke, R. Kruse and S. Seraffin, editors, *Symbolic and Quantitative Approaches to Reasoning and Uncertainty*, Granada: LNCS, vol. 747 (1993).

⁸ P.K. Schotch and R.E. Jennings, "On Detonating", in Priest *et al.*, *Paraconsistent Logic: Essays on the Inconsistent*, 306–327.

⁹ A half-century ago, in an impressive *tour de force*, McKinsey, Sugar and Suppes achieved an axiomatization of Newtonian mechanics. What if those axioms turned out to be inconsistent? How seriously should we be prepared to believe that physics would be destroyed? (J.C.C. McKinsey, A.C. Sugar and Patrick Suppes, "Axiomatic Foundations of Classical Particle Mechanics", *Journal of Rational Mechanics and Analysis*, 2 (1953), 253–272.

¹⁰ Peter Schotch, "Paraconsistent Logic: The View From the Right", *Philosophy of Science Association*, 2(1993), 421–429.

¹¹ Here is Schotch on this point: "In reasoning from an inconsistent set E, make things no worse than they already are, and in one respect at least make them better. So do *not* preserve triviality". "Paraconsistent Logic", See also Bryson Brown, "Simple Natural Deduction for Weakly Aggregative Paraconsistent Logics", in Dov M. Gabbay, editor, *Frontiers of Paraconsistent Logic*, Baldock: Research Studies Press 2000.

acquired in a systematic fashion. In axiomatic approaches, what the theorist discovers about sets resides in the demonstrative closure of the axioms. Suppose now that an inconsistency is derivable in such a theory. By the consistentists own lights, as evidenced by the historical reactions of Frege and Russell, no sentence of the theory is true as opposed to false. Every sentence and its negation is in the demonstrative closure of those axioms. Accordingly, a theory of sets is impossible.

This explains, well enough, our triple of figures. If no sentence of a theory of sets is true as opposed to false, knowledge of sets is impossible. If the theory cannot attain a knowledge of sets, we might just as well say that the inconsistency on which this paralysis rests has indeed blown the theory apart, or detonated it, or shut it down. Further, if the theory in question chanced to be the sole theoretical means of achieving a knowledge of sets (Frege, again), then these metaphors would apply not only to intuitive set theory but, by default, to any. Hence the very *discipline* of a mathematics of sets would have been shut down. In sections below, I shall return to this stronger claim. For now, we may take it that the metaphors have been adequately explained.

So, then, the catastrophe hypothesis fails for intuitive set theory. This matters. It constitutes the *founding insight of dialetheism*. If the presence of an inconsistency in a discipline does not shut it down, how plausible is it to suppose that its truth *would* shut it down?¹² What would matter, if anything would, is that set theory actually *harboured* the contradiction. We might imagine Descartes' God entering the picture, now having decided to make that contradiction true. Would this violate the Hippocratean requirement? It would not (unless accompanied by God's further decree to make free (and classical) use of this new truth as a premiss in further inferences!) This is telling. It allows the dialetheist to reject the catastrophe theory of Russell's contradiction and, in so doing, to mount the challenge to the consistentists of showing that the *truth* of that contradiction would have in fact made things worse for the mathematics of sets.

2. *A rivalrous triad*

The three doctrines that oppose consistentism's refusal to allow that *LNC* might have a true negation are themselves inconsistent with one another. Each of the three shares the view that some contradictions are true. Beyond that, they are pairwise incompatible. Against the dialetheist, trivialists and near-trivialists hold that many more contradictions are true than the dialetheist is prepared to countenance. Against each other, trivialists and near-trivialists disagree about whether the world is absolutely inconsistent. We have it from these tensions that each of the heterodox doctrines is open to attack on at least two fronts, as follows.

1. Dialetheism fails if either no contradiction is true or more than a few

¹² For a basic systems of dialethic logic see Robert Myer and Richard Routley, "Dialectical Logic and the Consistency of the World", *Studies in Soviet Thought*, 16 (1975), 1–25, and Graham Priest, "The Logic of Paradox", *Journal of Philosophical Logic*, 8 (1979), 219–241. A defence of true contradictions is developed in Graham Priest, *In Contradiction: A Study of the Transconsistent*, Dordrecht: Kluwer 1987, *Beyond the Limits of Thought*, Cambridge: Cambridge University Press 1995, and Bradley Armour-Garb, "Wrestling With (and Without) Dialetheism" *Australasian Journal of Philosophy*, to appear in 2004. Contradictory mathematics is examined by Chris Mortensen, *Inconsistent Mathematics*, Dordrecht and Boston: Kluwer 1995. For consistentist reservations, see Terence Parsons, "True Contradictions", *Canadian Journal of Philosophy*, 20 (1990), 335–382.

- are true.
2. Trivialism fails if either no contradiction is true or only some are true.
 3. Near-trivialism fails if either no contradiction is true or all are or few are.¹³

Accordingly, each has two additional “natural enemies” to keep an eye on, apart from consistentism. Dialetheists have to worry about trivialists and near-trivialists. Near-trivialists have to worry about dialetheists and trivialists. Trivialists have to worry about dialetheists and near-trivialists.

I want to examine the tensions and alliances between and among consistentism, dialetheism and trivialism. These are shifting rivalries of a certain complexity, to which I cannot do full justice in this exploratory note. Nevertheless I think that we might reasonably hope for the disclosure of some of the more basic features of the dialectical structure of these disagreements.¹⁴ Near-trivialism I reserve for another occasion.¹⁵

3. *Dialectics*

This would be a good place to say something about the “considerations” mentioned in the paper’s title. These are dialectical considerations. The word “dialectic” has had a long and ambiguous history, from ancient times¹⁶ to present-day concerns with conflict resolution.¹⁷ In one usage, ensuing from the writings of Marx and Lenin, “dialectic” refers to the (logically) contradictory structure of human history. In this sense, a *dialectical* logic would be a *dialetheic* logic, and indeed some logicians refer to such logics in just this way.¹⁸ This is not the meaning I intend here, for which the notion of a dialectical exchange is primary. Two or more adversaries are engaged in a *dialectical exchange* when they are reasoning with one another over some disputed issue. An enquiry into the structure of such arguments can be called “dialectic”.¹⁹

A standard form of dialectical exchange is one in which one party seeks to make a successful *ad hominem* argument against the other party. We should say at once that, as presently conceived, *ad hominem* arguments are a far cry from the fallacies of present-day logic textbooks in which they bear the same name. Here they have the character of

¹³ One could ask whether dialetheism fails when no contradictions are true but a few *might be* true, and whether trivialism fails when all contradictions are true, but some *might not be*. These are putative variations that need not concern us here.

¹⁴ It might also be observed in passing that the case for near-trivialism is a more difficult to make, strange as that might seem. Against this, it might be observed that the proposition that nearly all statements are true and untrue follows from the proposition that all statements are true and untrue. But this is not near-trivialism. Near-trivialism mimics the structure of dialeism, according to which some but not all contradictions are true. Near-trivialism embodies a similar qualification. Nearly all, but not all, contradictions are true.

¹⁵ John Woods, “Dialectical Considerations on the Logic of Contradiction: Part II”, to appear.

¹⁶ For Aristotle’s uses of the term see, for example, John Woods and Hans V. Hansen, “The Subtleties of Aristotle on Non-Cause”, *Logique et Analyse*, 176 (2001), 395–415.

¹⁷ See, again Frans H. van Eemeren and Rob Grootendorst, *Speech Acts in Argumentative Discussions*, Dordrecht: Foris 1985, and John Woods, “Dialectical Blindspots”, *Philosophy and Rhetoric*, 26 (1993), 251–265.

¹⁸ See, for example, Meyer and Routley, “Dialectical Logic and the Consistency of the World”.

¹⁹ Sometimes the plural is used: *dialectics*.

Locke's *ad hominem* arguments²⁰, which are a generalization of Aristotle's refutation arguments, discussed in *On Sophistical Refutations*, and of proofs-in-a-sense (also called *ad hominen*), discussed in the *Metaphysics*.²¹ The context in which *ad hominem* moves are made is one in which one party defends a thesis *T* from efforts of his opponent to discredit it. The attacking party seeks to draw consequences of the defender's defence of *T* that he (the defender) is moved to accept, even though doing so damages his original claim. As Locke puts it, the attacking party attempts to induce his opponent to give up his thesis by "drawing consequences drawn from his own principles or concessions".²² This is expressible economically. An *ad hominem* argument succeeds against a defender when the consequences it presents are, *by the defender's own lights*, more costly to accept than the costs of abandoning the thesis he had been attempting to defend.

Costs and benefits are construed as dispositions to hold or reject. A proposition is costly for an agent to the extent that he is not prepared to accept it. A proposition is beneficial for an agent to the extent that he is not prepared to reject it. Sometimes costs and benefits are reckoned up by calculation of the effort (or time, complexity, difficulty, or money) it would take to sustain a proposition one wanted to hold or to discredit a proposition one wants to reject. Doubtless, considerations of this sort often do affect an agent's dispositions to accept or reject. But the bottom line (to stay with the economics idiom) is that the cost of a proposition that induces one to reject it is represented by one's disinclination to accept that proposition's consequences, given their incompatibility with other propositions one is not prepared to abandon.

It is well to note that these bottom line costs and benefits are in the mind of the beholder; they are like subjective utilities. This is as needs be for disputes of a kind that lie open to the strong likelihood of mutual question-begging and of what Aristotle calls "babbling". (You babble when you simply repeat the thesis that your challenger has demanded that you defend.) If Harry's goal is to get Sarah to give up on a proposition to which Sarah is attracted, it is no good to focus on the hostility of this proposition towards propositions to which Harry is himself partial. Rather Harry must get Sarah to see that Sarah's proposition is hostile to other propositions to which *she* is more attracted than it. Giving subjectivity its head is the price we pay to subdue question-begging and to muzzle babble in disputes that lack the comforts of empirical checkpoints.

The idea of costs and benefits as criteria of a proposition's (or a theory's) acceptance or rejection is implicit in any pragmatic orientation to such things. Peirce at times is quite explicit in raising the issue of economic considerations, especially in the context of abductive inference; and it is well-embedded in Quine's "maxim of minimum mutilation".²³ Perhaps the most detailed analysis to date of cognitive economics is that of Rescher.²⁴

²⁰ John Locke, *An Essay concerning Human Understanding* (1690), bk. 4, chapter 17, § 19.

²¹ See here John Woods and Andrew Irvine, "Aristotle's Early Logic", in Dov M. Gabbay and John Woods, editors, *Greek, Indian and Arabic Logic*, volume 1 of the *Handbook of the History of Logic*, Amsterdam: North-Holland 2004, 27–100; especially 75–82.

²² Lock, *op. cit.*, p. 686 of the Nidditch edition of 1975, published by the Clarendon Press.

²³ Charles M. Peirce, *Collected Papers*, Cambridge, MA: Harvard University Press 1931–1958, 5.197, 5.599, 5.602, 7.220 W.V. Quine, *Philosophy of Logic*, Englewood Cliffs, NJ: Prentice-Hall 1970, p. 85.

²⁴ Nicholas Rescher, *Priceless Knowledge: Natural Science in Economic Perspective*. Lanham, MD: Rowman and Littlefield, 1996.

4. Decent logics

It might strike one as exceedingly odd that dialetheism is published in mainstream venues, and yet that the mainstream remains adamant in thinking dialetheism's principal claim to be dismissible out of hand. The oddity is more apparent than real. Dialetheic logicians find themselves in something like the position occupied by non-Euclidean geometers before the big spaces of relativity theory. Their heterodoxies were admired for their technical virtuosity rather than their truth. At their best, dialetheic systems are also technically adroit attainments. They meet the standards of what Joke Meheus calls "decent" logics.²⁵ A decent logic has a precise grammar, a proof theory, a model theory, and demonstrations of soundness and completeness. Meheus' characterization may well be a trifle tendentious, but it is not difficult to see what she means. A decent logic is a *bona fide* mathematical structure possessed of genuinely interesting metamathematical properties. Not every fancier of decency will want to include dialetheism in its ambit. But clearly some do; and they are consistentists for the most part.

Understandably enough, many dialetheists are dissatisfied with mere decency. They are troubled by the classification's implied anti-realism. People who admit dialetheic results into mainstream publications freely concede that, in various systems *S*, some contradictions are provable. They are true-in-*S*, but they are not true. For the irritated dialetheist this is damnation with faint praise. Truth-in-*S* is fine as far as it goes. But truth is better, and it is truth that dialetheists claim for their founding insight. Dialetheists find this hyphenation dismissive. They prefer to be realists about true contradictions.

Many — dare I say mainstream? — dialetheists stake their claim to true inconsistencies in a particular class of arguments. These are valid arguments with contradictory conclusions. Consistentists favour a uniform policy toward arguments of this type. They see them all as *reductios* of something in or presupposed by their premisses. Dialetheists are differently-minded. They see some of these arguments, but not all — in fact, hardly any — as sound demonstrations of surprising²⁶ (or startling or even shocking) truths. Consistentists and dialetheist are confronted by *Philosophy's Most Difficult Problem*.²⁷ This is the problem of distinguishing in a principled and non-question-begging way *reductio* arguments from proofs of something surprising. One of the distinctive virtues of proofs in general is their capacity to trump antecedent confidence to the contrary. The history of rigorous demonstration is rife with surprises once considered impossible, whether the existence of actual infinities, the nondenumerability of the reals, mechanical action at a distance, quantum non-locality, and so on. At their most interesting, proofs overwhelm our steadfastmost judgements of impossibility. Why, then, should we not recognize this trumping potential in application to the steadfast conviction that true contradictions are impossible?

²⁵ In a talk to the Abduction Symposium at the Conference of the Cognitive Science Society, in Chicago, August 2004.

²⁶ In German, *befremdlich*. See Ernst Mally, *The Basic Laws of Ought*, Graz: Leuschner and Lubensky 1926, 20–34.

²⁷ John Woods, *Paradox and Paraconsistency: Conflict Resolution in the Abstract Sciences*, Cambridge and New York: Cambridge University Press 2003, pp. 14–16 and 76–78.

Consistentists could be forgiven for thinking this rhetorically excessive. It is perfectly true that ours is an intellectual tradition in which appearances to the contrary are often corrected by proof. But Philosophy's Most Difficult Problem is a problem for the very institution of proof. It is the problem posed by the difficulty of distinguishing between good proofs and merely good-looking proofs, and between bad proofs and merely bad-looking proofs. Consistentists think that they have a solution to a significant part of this problem. They classify all valid proofs whose conclusions are contradictions as *reductios*, and they do so on the grounds that the truth of the purported conclusions would violate *LNC*, which itself is inviolable. It is a principled answer to the two questions. It explains why a proof cannot trump the conviction that contradictions can't be true. And it explains why paradoxical proofs cannot be construed as sound demonstrations of surprising (or even madly startling) conclusions.

Even so, the dialetheist's own position is that, by and large, contradictions aren't true. This brings him into a near-alliance with the consistentist. The consistentist's position is that paradoxical proofs are unsound. But this is also the dialetheist's *default position*.²⁸ What separates the two is the question of how to interpret the syntactic structure of *LNC*. The consistentist sees it as a law which licenses the *generalization*

- contradictions are never true

whereas the dialetheist sees it as the *generic claim*

- contradictions aren't true.

The consistentist sees the generalization as a universally quantified conditional. The dialetheist sees the generality on the model, not of universally quantified conditionals, but rather on that of "Tigers are four-legged". Consistentists want it ruled out in advance that there might be exceptions. Dialetheists are prepared to attend to a purported exception on the merits of their case.

Dialetheists are correct in noticing, both in logic and everyday reasoning, that the consistentist's attitude to *LNC* is deeply entrenched, to the point of dogmatism. Dialetheists want to question this favouritism. It is the pragmatic thing to do. In particular, dialetheists have a stake in questioning the following linked pair of consistentist claims:

1. True contradictions can't exist, since this would violate the Law of Non-Contradiction, which is a first principle, i.e., a principle which is true and neither requires nor admits of proof.
2. *Therefore*, the damage done by paradoxical proofs cannot be to the Law of Non-Contradiction, but must be to something else.

In the section to follow we shall show that (1) lies open to challenge, and in the section following it, that this means that there is room in which to challenge (2) as well.

²⁸ See here Diderik Batens, "Inconsistency-Adaptive Logics", in E. Orłowski, editor, *Logic at Work: Essays Dedicated to the Memory of Helena Rasiowa*, Heidelberg: Physica Verlag Springer 1999, 445–472 and Diderik Batens, "A Survey of Inconsistency-Adaptive Logics," in D. Batens, C. Mortensen, G. Priest and J.P. van Bendegem, editors, *Frontiers of Paraconsistent Logic*, Baldock: Research Studies Press 2000, 49–73.

5. Dealing with Philosophy's Most Difficult Problem: An Aside

In previously published work, I have examined the dialectical structure of Philosophy's Most Difficult Problem²⁹ This is not the place to reprise that discussion in any detail, but perhaps a brief summary would be appropriate for our present task. To begin with, it is necessary to emphasize that my strategies for the solution of instances of Philosophy's Most Difficult Problem (*PMDP*) are *dialectical* in character, rather than, as we might say, *probative*. I inherit this dialectical orientation from Aristotle and Locke. A solution for a *PMDP* is a dialectical one when it is accepted by the solver's *interlocutor*. When this happens it is not at all necessary that the solver share his interlocutor's beliefs or commitments. Moreover, a *PMDP* solution is *economic* when the interlocutor's capitulation is governed wholly by what propositions he is willing or not willing to give up. In other words, an interlocutor's acquiescence is entirely a matter of his own estimate of the bottom line. Even so, we can with reasonable confidence identify some costs in advance of a given interlocutor actually proclaiming or acting on them:

- question-begging is a cost
- babbling is a cost
- being committed to a proposition one deeply believes to be false is a cost.
- being committed to a proposition that requires a massive overhauling of one's belief-set is (often) a cost.

It is, of course, hugely important that a dialectical resolution of a *PMDP* not be confused with a proof of the solution's winning proposition. (It is not for nothing that Aristotle took care with this point, insisting that although any such solution is a "proof *ad hominem*", it is not a "proof in the full sense".)³⁰ Given their capacity for probative shortfall, one might wonder what the value of dialectical solutions might be. Even at their probative weakest, a successful dialectical attack upon an opponent establishes the opponent's inability, by his own lights, to sustain a defensible case for this own view. But, in the general case, dialectical adversaries are also members of the same (or anyhow overlapping) *epistemic* communities. What this means in practice is that, by and large, when dialectical adversaries interact with one another, the views they exchange are those the one holds as true and advances with the anticipation that the other will also hold it as true. Accordingly, a dialectically successful resolution of a *PMDP* may well coincide with a probatively successful resolution of it.

In *Paradox and Paraconsistency*, I proposed that the two dominant strategies of *PMDP*-resolution are the method of *surrender* and the method of *reconciliation*. Surrender is achieved when the attacker elicits concessions from his opponent that would discredit the thesis he is defending if they were actually accepted by him, and are indeed accepted by him. An historically interesting example of surrender is the Lewis-Langford proof of the consistentist's claim that negation-inconsistency generates absolute inconsistency. It is a simple conditional proof, whose proof rules are merely simplification, addition and disjunctive syllogism.³¹ The proof formed the basis of a (rhetorical) challenge to those who disliked *ex falso quodlibet*: "Which of these rules are

²⁹ *Paradox and Paraconsistency*, pp. 14–16, 27, 76–78, 80, 90, 126 and 156.

³⁰ *Metaphysics* K5 1062^a 2–3.

³¹ C.I. Lewis and C.H. Langford, *Symbolic Logic*, New York: Dover 1932, p. 252.

you prepared to give up?”³² The second method of resolution — what I call reconciliation — involves the search for different things for the adversaries’ competing intuitions to be true *of*. Keeping with the historical example of the rivalry between strict implicationists and relevantists about the *ex falso* issue, the reconciliationist strategy proposes that what *ex falso* is true of is the entailment relation, and what *ex falso* is false of is the operation of inference, and that entailment and inference are sufficiently different things to warrant this kind of settlement.³³ Although the strategies of both surrender and reconciliation can be pressed with probative intent, the bottom line is that they stand or fall on the parties’ *agreement*. They are *au fond* dialectical strategies.

6. *LNC as a first principle*

Aristotle famously held that the Law of Non-Contradiction is *primus inter pares*, the most imperious of first principles, neither requiring nor admitting of demonstration.³⁴ In our own day, so the story goes, Richard Sylvan³⁵ once challenged David Lewis to prove the Law. Lewis refused to be drawn, pointing out in the manner of Aristotle that no such proof was possible, and that this was a fact that reflected no discredit upon it.³⁶ Consistentists and dialetheists don’t disagree about everything, of course. They agree that *LNC* cannot be proved. But they also disagree about the matter on which they agree. Some consistentists think that the reason that it is not possible to prove that there aren’t any true contradictions is that this *LNC* is a first principle. Dialetheists think that the reason is that the *negation* of *LNC* is true, hence that *LNC* is not a first principle.

In what we might call the *Lewisian* approach to *LNC*, we have it that

- (1) The proposition that there aren’t any true contradictions is true and neither requires nor admits of proof.

If (1) is true, it disarms Sylvan’s challenge. Short of begging the question, one could not ask a consistentist to prove *LNC*. But is it not open to Sylvan or any other dialetheist to ask whether (1) *is* true? Is it not also legitimate to ask, if (1) is true, what makes it so? Might we not wish to know what a proof of (1) might be like? A number of possibilities come to mind.

³² As the record shows it was a challenge which consistentist-minded logicians found unanswerable. Steadfast relevantists, however, swallowed hard and showed disjunctive syllogism the door.

³³ See here John Woods, “The Relevance of Relevant Logic”, in Jean Norman and Richard Routley, editors *Directions in Relevant Logics*, Dordrecht and Boston: Kluwer 1989, pp. 77–89.

³⁴ There is however some question as to whether Aristotle thought that he had *formulated LNC* adequately. After having asserted of this “most certain of all” principles at *Metaphysics* Γ 1005^b 19–23 that it guarantees “that the same attribute cannot at the same time belong and not belong to the same subject in the same respect”, he goes on to say that “we must presuppose, in the face of dialectical objections, any further qualifications which might be added.” At *Metaphysics* 1006^a 1–^b 34 and 1010^b 11, Aristotle expressly considers the possibility of true contradictions. He allows that if *LNC* does not hold of all cases, “the exceptions will be agreed upon” (1008^a 10–11).

³⁵ *Ne’* Routley.

³⁶ This episode is reported by Roy Sorenson in a talk to an Author Meets His Critics symposium (John Woods, *Paradox and Paraconsistency*), Eastern Division, American Philosophical Association, Washington, DC, December 2003. As is clear from *Metaphysics* !, Aristotle was not so resolute. For a recent discussion of *Metaphysics* !, 1003^a21–1–2^b34, see Graham Priest, “‘To be *and* not to be — That is the Answer’. On Aristotle on the Law of Non-Contradiction”, *Philosophiegeschichte und Logische Analyse*, 1 (1998), 91–130.

- (i) A proof of a proposition could be a philosophical argument that justifies it.
- (ii) A proof of a proposition could be a deduction from logical rules or first principles (or axioms).
- (iii) A proof of a proposition could be an argument from principles embodying epistemically prior concepts.

A standard way in which to offer a proof of (1) in sense (i) is to claim that the proposition that *LNC* is a first principle is *analytic* and known *B priori*. Perhaps this is so, but it is dialectically unavailing to say so. The dialetheist's position is that *LNC* fails. If it does fail, then *LNC* cannot be a first principle, and (1) cannot be true. But if (1) isn't true, it cannot be either analytically true or known (or knowable) *B priori*. Any such claim is dialectically unavailing. It is unavailing because it begs the question.³⁷

The alternative suggestion that (1) is provable in sense (ii) fares no better. Let Δ be any deduction of which (1) is the terminal member. (1) is the proposition that *LNC* is true and neither requires nor admits of proof. Δ fails unless it proves that *LNC* is true. Δ also fails unless it proves that *LNC* can't be proved. So, under the obvious Tarskian equivalence, in which *A* is true iff Δ fails.³⁸

If (1) were subject to a proof in sense (iii), the objection against the sense (ii) option could be re-pressed. A proof in sense (iii) resembles a proof in sense (ii) except that the lines preceding the conclusion are conceptually prior or epistemically more primitive than (1) itself.³⁹ There is in these qualifications ample occasion for puzzlement and contention. But it doesn't matter. If there is a proof in sense (iii) of (1), there is proof that *LNC* is true and that *LNC* cannot be proved, which, given that same Tarskian equivalence is fatal.

7. *Privileging proposition (1)*

Have we given too short a shrift to the analyticity defence of *LNC*? Might it not be argued that one achieves a philosophical justification of *LNC* not by merely *asserting* its analyticity but by explaining how it comes to *be* analytic? Could we not say that *LNC* is true by the meaning of negation and conjunction? Certainly that is a widely held view. And it is certainly correct for consistentist—indeed, classical — interpretations of \neg , \wedge (and \top and \perp).⁴⁰ But this is not the place for this argument. What we are concerned with here is not whether there is proof (or philosophical justification) of *LNC*, but rather with whether there is a philosophical justification of (1), the proposition that asserts the proposition that there aren't any true contradictions is true and neither requires nor admits of proof. Suppose that the present proposal is sound, that *LNC* is true, rather than false, by virtue of meanings. But this cannot serve as a philosophical justification of (1). For here too, if it did establish the truth of (1), it would establish the truth of *LNC*; but it would also establish the truth of the claim that *LNC*'s truth can't be established.

³⁷ It bears emphasizing that the analytic *B priori* defence fails *dialectically*. It fails even if epistemology's diminished confidence in analyticity and apriority over the past decades has been misplaced.

³⁸ Another reason to dislike the present suggestion is that it embodies a sense of "deduction" according to which *LNC* is provable from itself. But this is the lesser objection of the two.

³⁹ Reminiscent of Aristotle's conception of demonstrative syllogisms.

⁴⁰ Here is Quine on this point: "They [the dialetheists] make negation unrecognizable." (*Philosophy of Logic*, p. 79 ff.)

Even so, taken apart from its role as a justification of (1), the meaning defence of *LNC* itself is problematic. It lies open to two objections. One is that *LNC* does not itself announce a favouritism for classical interpretations of \neg , \vee , T and F. (Intuitionist logics uphold *LNC* but give classically deviant readings of \neg , \vee , T and F.) The other is that the empirical diachonics of linguistic practice puts paid to any notion that the extension of a term is strictly fixed by its meaning. (There was a time in the history of English when the meaning of “deer” — synonomous with “animal” — stopped sanctioning the application of that term to wolves and rabbits). In the real world, meaning often lags behind reference, *pace* the Fregeans. Even if this is wrong, a prior complaint stands. Rooting the truth of *LNC* in the meanings of \neg , \vee , T, and F begs the question against the dialetheist, for whom the opposite is true.⁴¹

It might strike us that the reason that it is so difficult to find a sense in which (1) responds to a proof that is both sound and dialectically availing is that, like *LNC*, (1) *itself* neither requires nor admits of proof. That is to say, perhaps it is the case that

(2) The proposition that the proposition that there aren't any true contradictions is true and neither requires nor admits of proof is true and neither requires nor admits of proof.

Equivalently, just as (1) says that

(1') *LNC* is a first principle

(2) says that

(2') That *LNC* is a first principle is a first principle

So far as I know, no consistentist has openly subscribed to (2'). But it will do harm to see, briefly, why it won't do.

If (2') is true then (1') is true; that is to say, the truth of (2') proves the truth of (1'). But if (1') is true, it follows that *LNC* is true. So if (2') is true, it proves *LNC*. Hence (1') is not true.

This is interesting. Not only does the claim that the statement that *LNC* is a first principle is itself a first principle harbour an inconsistency, every statement of like form does; that is, every statement in the form

(3') That *P* is a first principle is a first principle

is inconsistent. This is worth flagging.

- The first-principle predicate does not admit of consistent iteration.

We have it, then, that (2') is false. So the proposition that *LNC* is a first principle is itself either untrue, or requires a proof, or admits of a proof. Of the three, we might

⁴¹ Intuitionist, many valued and (in some versions) quantum logics all pledge to *LNC*, and each could justify these pledges by appeal to quite different meanings of \neg , \wedge , T and F from each other, as well as from classical logic. We have it, then, that those who fancy *LNC* are drawn to interpretations of \neg , \vee , T and F that give them their head on this score. Would this not leave room for the dialetheist to seek for interpretations of \neg , \wedge , T and F that caters for his own view of *LNC*? Against this, it might be argued that there is no *tenable* interpretation of \neg , \wedge , T and F that would allow *LNC* to admit of true instances of its negation. Perhaps this is right; but the consistentist cannot have it that what *makes* it right is that the negation of *LNC* can't have true instances. Saying that would beg the question against the dialetheist. So the question before us is in all essentials whether the dialetheist logician has produced for his logic anything deserving of the name of semantics. The answer is, “Of course he has”. (See section 19 below).

suppose that the consistentist would likely pass on the first two and settle for the third.⁴² But, again, if (1') admits of a proof, it would prove that

- i) *LNC* does not admit of a proof
- ii) *LNC* does not require a proof
- iii) *LNC* is true.

But, by that obvious Tarskian equivalence, we have it again that if (1') admits of a proof, so does *LNC*; hence that (1') is false and thus does not admit of a proof.

Where do we now stand? In none of the senses of proof examined so far is there a proof of (1') that works against the dialetheist. Nor will it do to inoculate (1') against dialetheical challenge by claiming first principlehood for *it*. None of this is remotely close to being decisive against the consistentist. But it does put non-trivial (no pun) pressure on the consistentist's standard response to the dialetheist's challenge: Since *LNC* is a first principle, the damage done by the Russell paradox (and the Liar and the like) must be damage to something other than *LNC*.

This is a dialectical point. The consistentist and the dialetheist are involved in a disagreement. It is a disagreement of a particularly unsettling kind, made so by the absence of empirical checkpoints that settle the matter. It is a disagreement of a kind wholly typical of those that crop up in the foundations of mathematics and logic, and more generally in most of philosophy. The dialetheist has challenged the claim that *LNC* is unchallengeable. The Lewisian response is that *LNC* is a first principle. The dialetheist has challenged this further claim, and as yet to receive an adequate response to it. A dialectical contention is like a tango. It takes two to dance one. The Lewisian is free to call a taxi and simply go home. He is under no obligation to tarry with his opponent. This hardly needs saying. But if the Lewisian decides *not* to go home, and if instead he agrees to give the dialetheist a hearing, the point that he hears is a dialectical one. The dialetheist is trying to show that the consistentist's standard response to the dialetheist's diagnoses of the Russell and the Liar is unsatisfactory on dialectical (indeed broadly Lockean) grounds.

8. Alternative defences of *LNC*

(1') is not the only defence of *LNC*, perhaps not even the dominant one. Two others come immediately to mind.

(4) *LNC* is a regulative principle of inference (and of experience as well).

(5) *LNC* is much less costly to retain than to give up.

It is clear that (4) and (5) bear some affinity to one another. They both forward economic considerations. (4) sounds a pragmatic theme. *LNC* is useful, indeed necessary, in keeping thought and action on track. (5) is more avowedly economic. *LNC* is too precious to give up. It is well to take (4) and (5) as a seemingly caution. They counsel against making too much of the difficulties that face (1'). We may agree that those difficulties to set up *LNC* as a legitimate target of dialetheic challenge. But it may well be that, via (4) or (5) or both, the challenge can aggressively be met. But perhaps it will also be agreed that what I've been calling the consistentist's standard objection to true conditions has been

⁴² If he gave up on the first principlehood of *LNC*, a Lewisian refusal would in turn lie open to dismissal out of hand. If *LNC* requires a proof, a Lewisian refusal is all the more embarrassing.

compromised by the difficulties that attend the first-principle defence of *LNC*. That being so, we shall now turn our attention to the dialetheist's further claim, viz.

(6) The damage done by paradoxes such as the Russell and the Liar are to *LNC*.

If (6) is right, *LNC* is a compromised principle. By dialetheic lights, it is a principle that admits of true instantiations of its negation. By those same lights, "damage" is not quite *le mot just*. "Damage by *classical* lights" would be a better way of saying it.

Dialetheists think that the proofs of the Russell and the Liar are sound demonstrations of something surprising. But it is dialectically unavailing to say so to anyone who holds that those same proofs are *reductios* of something in or presupposed by their respective premiss-sets. Let us say again that the dialetheist and the consistentist have been landed in a dialectical exchange. They are free to think what they will and say what they think. But they must not suppose that whatever they may think or say will be dialectically availing, *even if true*. In particular, they are not free to beg one another's questions.

How might the dialetheist now proceed? Against the consistentist's defence (3), that *LNC* is a regulative principle of thought and action, he is able to make two responses. One we have met with before. It is that if the Russell paradox blows set theory apart, set theory would have blown apart long since. But it didn't (and hasn't). So the catastrophe hypothesis cannot be true in this, or any requisitely similar, instance. The other response links to the first. It is perfectly possible that *LNC* is indeed a regulative principle of thought and action. But for this to be so, it is hardly necessary that *LNC* be true (only). Nothing in dialetheism requires that the *regulative* force of *LNC* be called in to question (This will prove a fateful come-uppance in section 13 below.)

This leaves us with (5), the consistentist's economic defence, by which *LNC* is too precious to give up. "Too precious" here implies "too costly."

9. Economics

It is salutary that the contention between dialetheists and consistentists over (4) is so openly an economic one. It is economic considerations that drive the ebb and flow of dialectically effective management of intractable disagreement, and that provide for whatever prospects there may be for satisfactory resolution. Again, it bears on this in a fundamental way that these are disagreements that occur in the absence of empirical checkpoints. These are not "white coat" disagreements. In such contexts, resolution is by agreement, and agreement is got by negotiation. Absent consensus, the disagreement stands unrepaired. Economic considerations help situate our disagreement in a philosophical context championed by Quineans, among others. It is a pragmatic context in which the principles of logic (or anything else) stand or fall on the ultimate good they do in the big picture of science. The good that they do Quine measures by the maxim of minimum mutilation.⁴³ These days, there is nothing new about this pragmatism about

⁴³ Here is Quine in *Philosophy of Logic*: "Logic is in principle no less open to revision than quantum mechanics or the theory of relativity. . . . If revisions are seldom proposed that cut so deep as logic, there is clear enough reason for that: the maxim of minimum mutilation . . ." (p. 100).

logic. What *is* new are the lengths to which dialetheists are prepared to take the slack that pragmatism cuts them.⁴⁴

The economic argument to which the Russell paradox gives rise is a dispute about which the is the more costly of the following pair of options.

Option one: to reconfigure set theory in such a way that the contradiction is expunged without noticeable damage to the theory's capacity to provide a mathematically robust account of sets.

Option two: to leave the contradiction in place, and let set theory run on, in the manner of its running before the contradiction was discovered.

Here the consistentist favours option one because he thinks classically about option two. He thinks that if the paradox is left in place and is true, then set theory has indeed detonated. Its every sentence is true. Against this, the dialetheist proffers the empirical observation that pre-paradox set theory didn't detonate, and he concludes from this that one or other of two further possibilities is true.

- classical consequence is made defective by *ex falso* quodlibet.
- set theoretic inferences are (obviously) non-classical.

These further possibilities, give rise to others --- notably, that the dialetheist has lumbered himself with the necessity to re-do the logic of consequence.⁴⁵ Since this something that he is already keen on doing (since dialetheists are also paraconsistentists), the dialetheist can hardly take this as an objection. Even so, it shouldn't be overlooked that this is an ambition attended by costs of its own.

It is not my intention to examine the economic give-and-take over *LNC* beyond this point, still less to propose a final adjudication. My aim so far has been to lend credence to some telling *dialectical* points.

- Contentions about *LNC* are not doomed to deadlock.
- The consistentist has no dialectically tenable grounds on which simply to deny dialetheism a hearing.
- It is not given *B priori* that dialetheism could not produce a winning cost-benefit argument against consistentism.⁴⁶

This is far from giving the nod to dialetheists. My plea has been only that they not be given the hook.

10. Externalities

The actual history of set theory post-Russell plays out with a pinching irony. Had there been dialetheic apologists on the scene in 1902, it is conceivable that that option two would have been exercised. Had anyone been there to think about it, people might well have been reassured by its attractive conservatism. As events turned, no one did

⁴⁴ "One effect of abandoning" the immunity-from-overthrow-position towards logic "is a shift toward pragmatism." (Quine, "Two Dogmas of Empiricism", *The Philosophical Review*, 20 (1951), 20–43).

⁴⁵ Some investigators of these possibilities argue that the truth of the second does nothing to support the first. Their argument would stand if, to put it loosely, the principles of logic were not, just so, principles of belief-modification. See here Gilbert Harman, *Change in View*, Cambridge, MA: MIT Press 1986, chapters 1 and 2, and John Woods and Douglas Walton, *Fallacies: Selected Papers 1972–1982*, Dordrecht and Providence: Foris 1989, chapter 1.

⁴⁶ But see Woods, *Paradox and Paraconsistency*, pp. 192–195.

think about it. And option one was exercised *faute de mieux*.⁴⁷ This matters. By the time the dialetheic option entered the mainstream, mathematicians had had a generation to get used to option one. The result is *ZF* (or *ZFC*). It is now routinely taken as the best account of the intuitive notion of set. Whether or not it is, *ZF* is a deeply dug-in and flourishing orthodoxy. Flourishing orthodoxies are standing discouragements of rebellion. The costs of their displacement run high.

We may take it that the dialetheist will not succeed in toppling *ZF* in the foreseeable future. A setback no doubt, would be this decisive against the dialetheist's option two? The answer is "No". If *ZF*'s hegemony arose, at least in part, from the historical contingency of there being no effective dialetheic force to challenge it at birth, it may be said that its status as a cost-benefit winner derives in part from what economists call an "externality". To the extent to which this is so, the dialetheist has room to surmise that *had* the dialetheic option been "on the table" in 1902 and following, the history of set theory might well have been different; that is to say, dialetheic. Were he to be correct in his surmise, a plausible conclusion would drop out. The triumph of *ZF* was, to some extent, an economic anomaly occasioned by a deficit of information.⁴⁸

11. The importance of the Liar

Dialetheism is not a one-trick pony. It does not stand or fall on the success of its preferred treatment of the Russell paradox. Other paradoxes — notably the Liar — get the same rise from the dialetheist. Here, too, there is ready occasion to debate the merits of a similar pair of options. One is to expunge the Liar contradiction and rejig semantics accordingly. The other is to let well enough alone, and to build one's semantics around the contradiction. In this case, it is less clear that the second option is the costlier of the two. The difficulty, again, is that dialetheic semantics has been beaten to the punch historically, although not as decisively as in the case of sets.⁴⁹ Comparatively speaking, dialetheic semantics is not as well-filled out as the Tarskian orthodoxy or its "intuitive" variants in the manner of Kripke, Herzberger and Gupta, to name just three.⁵⁰ This is not

⁴⁷ More irony still: by 1902 Moore and Russell had effectively buried idealism, which is one place in which the dialetheic option might have found a home.

⁴⁸ Against the present suggestion, one of this journal's anonymous referees writes as follows: "Set theoretic intuitions were not sufficiently strong to allow mathematicians to proceed informally just ignoring the contradiction. I further doubt whether a formal system in which the contradiction was allowed to stand but was contained, could have been a serious competitor to Zermelo set theory in terms of smooth working etc." I am grateful for these remarks. They give me pause. Perhaps a more secure and historically determinate case is that of the inconsistency of the calculus. The inconsistency was pointed out by Berkeley. The calculus of Newton and Leibniz treated the infinitesimals as real numbers of a certain kind: They were greater and not greater than zero. Berkeley's complaint was acknowledged and continues to be so to this day in standard texts. No one took the inconsistency to disable the calculus. Mathematicians just doldiered on, and mechanics flourished. Now, of course, ascriptions to reals, which result in inconsistency, evade the inconsistency when made to the hyperreals. Since the inconsistency did not disable the calculus, shall we assume that, all along, talk of infinitesimals was implicitly talk of hyperreals, never mind that nobody, until Robinson, would have heard of the hyperreals? (Abraham Robinson, *Non-Standard Analysis*, Amsterdam: North-Holland 1966).

⁴⁹ Even the popularized version of Tarski's diagnosis and cure, "Truth and Proof" appeared in *Scientific American* as early as 1942.

⁵⁰ Saul A. Kripke, "Outline of a Theory of Truth", *Journal of Philosophy*, 72 (1995), 690–716; Hans G. Herzberger, "Notes on Naïve Semantics", *Journal of Philosophical Logic*, 11 (1982), 62–102; and Anil

the matter that matters most here. For one thing, it is always open to the dialetheist to re-deploy his economic externality complaint. What matters about the dialetheist's preferred treatment of the Liar, is not the treatment but the nature of the Liar itself. The problem is twofold. One is that more paradoxes exist than one can shake a stick at.⁵¹ Yet dialetheists think that only a few of these are sound proofs of something surprising. This faces the dialetheist with the essential task of distinguishing in a principled way those sound few from the reductive many.

The second difficulty is a particular case of the first. By dialetheic lights, the Liar delivers the goods for dialetheism. Dialetheists also think that the so-called semantic paradoxes should be given like treatment.⁵² The Curry paradox is a semantic paradox. If sound, it delivers the goods for trivialism, the doctrine that everything is contradictory. Dialetheists abhor trivialism at least as intensely (and confidently) as consistentism abhors dialetheism. But if the semantic paradoxes are to be treated alike, how can we have it that the Liar is a sound proof of a contradiction and the Curry is a *reductio* of something in or presupposed by its prior lines?

My purpose is not to adjudicate the claim that if the Liar establishes dialetheism, then the Curry establishes trivialism.⁵³ (However, I think that the claim is true. Readers may wish to consult informal versions of the two proofs in the *Appendix*. Perhaps they will find grounds for their differential evaluation that I, and others, fail to see.) I want instead to turn to a different matter. I want to press the dialetheist on his *resistance* to trivialism. I shall turn to this issue in the section after next.

12. A diagnostic interlude

The dominant consistentist reaction to the Liar is Tarskian. But all the standing variations, including Tarski's own, are responses to a diagnosis other than the one actually made by Tarski in "The Concept of Truth in Formalized Languages". The two dominant "Tarskian" diagnoses are that the Liar paradox shows that

- the intuitive concept of truth is inconsistent.
- the semantics of natural languages is inconsistent.⁵⁴

Neither of these matches Tarski's words. Tarski is troubled by the semantic closedness of natural languages, or by what he calls the "universality" of "colloquial" languages. Tarski writes,

Gupta, "Truth and Paradox", in Robert L. Martin, editor, *Recent Essays on Truth and the Liar*, Oxford: Clarendon Press 1984, 175–236. Kripke and Herzberger are also reprinted in Martin. For more recent treatments of the Liar, see J C Beal, editor, *Liar's and Heaps: New Essays on Paradox*, Oxford: Clarendon Press 2003, and Graham Priest, J C Beal and Bradley Armour-Garb, editors, *The Law of Non-Contradiction*, Oxford: Clarendon Press 2004.

⁵¹ See the extensive survey in Michael Clark's *Paradoxes From a to z*, London: Routledge 2002, in which we find 75 entries. Not all of these are arguments whose conclusions are contradictions. Even so the number of antinomous cases is large.

⁵² See Bradley Armour-Garb, "Wrestling With (and Without) Dialetheism".

⁵³ This is taken up in John Woods and Bradley Armour-Garb, *Dialetheism Pro and Con: Could Contradictions Be True?* forthcoming in 2006.

⁵⁴ Among other responses, perhaps most dominant is one that finds that "This statement is untrue" expresses no statement. See, e.g., Andrew Irvine, "Gaps, Gluts and a Paradox", in Bruce Hunter and Philip Hanson, eds., *Return of the A Priori*, *Canadian Journal of Philosophy*, Supplementary vol. 18 (1993), 273–299. But see § 13, below.

If we analyze this antinomy [i.e., the Liar] . . . we reach the conviction that no consistent language can exist for which the usual laws of logic hold.⁵⁵

Lines previously, he makes the same point.

These antinomies seem to provide a proof that everyday [i.e., “colloquial”] language which is universal in the above sense, and for which the normal laws of logic hold, must be inconsistent.⁵⁶

Tarski is a consistentist. It is gospel for consistentists that consistency is a condition on existence. In the quoted passages, Tarski twice says that every natural language is inconsistent. But if nothing inconsistent exists, Tarski should have concluded that natural languages are a massive illusion, that their inconsistency precludes their existence. Had this actually been Tarski’s diagnosis, it would have handed dialetheists a clear cost-benefit winner. The non-existence of human language is unimaginable. If true, it exacts costs in comparison to which the costs of an occasional piddling contradiction are trifling. Although some investigators in the Tarskian tradition see Tarski’s solution as a hierarchical reconfiguration of natural languages, Tarski himself emphasized the necessity to relocate the truth predicate to formalized languages. Tarski, like Frege⁵⁷ and Peirce⁵⁸ before him, simply gave up on natural language for the purposes of logic and mathematics. Tarski gives a third characterization of the moral of the Liar:

If these observations are correct, then the very possibility of a consistent use of the expressions “true sentence” which is in harmony with the laws of logic and the spirit of everyday language seems to be very questionable⁵⁹.

In comparison with the prior two, this may seem a much soft-pedalled diagnosis which comports well with the Tarskian mainstream’s assessment of the Liar. But it does not comport well with Tarski’s own abandonment of the natural languages in favour of the formal. If, as Tarski asserts in the third quotation, the damage done by the Liar is to the intuitive conception of truth, one might think that this is markedly slighter diagnosis than the one that implies the non-existence of human languages. But, even here, there is a difficulty. If it is the predicate “is true” that the antinomy shows to be inconsistent, there is no such thing as truth. Accordingly, nothing is true, including every sentence of Tarski’s “On the Concept of Truth in Formalized Languages”.

This leaves the suggestion that the most that the Liar shows is that a certain approach to the semantics of English and the other human languages is defective.⁶⁰ This is the theory that allows unbridled semantic self-reference. Nothing in the Tarskian oeuvre supports so benign a reading of Tarski’s words. But, never mind, it is a reading of the Liar that is not lightly dismissible.

How, then, do these readings position the dialetheist? The first, the reading that dare not speak its name, creates a cost-benefit winner. It is a rendering that forwards a

⁵⁵ Tarski, “The Concept of Truth in Formalized Languages”, pp. 164–165.

⁵⁶ *Idem.*

⁵⁷ Frege held a thesis of radical non-bivalence, in which no sentence of a natural language had a truth value. If, notwithstanding, sentences of English were handled classically, English on Frege’s conception of it would dance to trivialist presumptions.

⁵⁸ Peirce thought that the very idea of a logic of natural language was ludicrous.

⁵⁹ *Idem.*

⁶⁰ Or that a certain class of first order theories is defective. Cf. “Tarski showed that no consistent first-order theory with a good grip on its language’s syntax could be semantically closed.” (Stephen Yablo, “New Grounds for Naïve Truth Theory”, in Beal, *Liar and Heaps*, pp. 312–330; p. 312.

scepticism as corrosive as any that moved the likes of Cratylus. Next to it, a scant sprinkling of true inconsistencies is small beer.

The judgement that the fault lies in the inconsistency of truth fares little better. It requires us to concede that every utterance anyone has ever made in his own tongue or any other is untrue. Next to that, the truth of the Liar sentence is not much to write home about.

The third interpretation leaves the dialetheist no worse off than does the Russell paradox. If what the Liar shows is that a certain kind of semantic theory for a certain class of languages is defective, just as the Russell shows only that a certain theory of sets is defective, then dialetheists think it is reasonable to ask, “How damaging is this?” The classical answer is that the damage is awful, that semantical theories of universal languages blow apart. But, as with the case of intuitive set theory, they didn’t in fact blow apart. The catastrophe hypothesis is wrong for sets. It is also wrong for semantics.

It is interesting to conjecture why Tarski’s own readings of the Liar are at once greatly more damaging than anything that comes down to us in the consistentist aftermath and, at the same time, are readings whose crippling ferocity Tarski never seemed to notice. The Russell paradox occasioned the same diagnostic confusion. Frege drew an inference resembling the strong Tarskian interpretation: *There are no sets*.⁶¹ Russell temporized. There are no *real* sets, but something set-like can be recovered by nominal definition.⁶² Notwithstanding the utter direness of Tarski’s actual diagnoses of the Liar, his method of treating it suggests something a good deal less awful. Far from showing the impossibility of human language or the utter non-existence of truth, what Tarski gives us is, in effect, a new semantics for (a successor) of truth in which (so far as we yet know) the contradiction doesn’t reappear. Tarski’s semantic hierarchy resembles Russell’s type theory for sets. Nor is this the only resemblance. Russell’s diagnosis of the paradox of sets is equally dire, and his mode of repair is similarly underplayed. Russell thought that the paradox that bears his name destroyed the very *concept* of set. This, too, was Frege’s view. Frege retired from the philosophy of arithmetic. Russell muddled through. He re-defined sets.

It is necessary to realize that definition, *in mathematics*, does not mean, as *in philosophy*, an analysis of the idea to be defined into constituent ideas. This notion, in any case, is only applicable to *concepts*, whereas in mathematics it is possible to define terms which are not concepts. (*Principles of Mathematics*, p. 27. Emphasis added)

Moreover,

Of the three kinds of definition admitted by Peano — the nominal definition, the definition by postulates, and the definition by abstraction — I recognize *only the nominal*. (*Principle of Mathematics*, p. 112. Emphasis added.)

⁶¹ Gottlob Frege, “Letter to Russell”, in Jean van Heijenoort, ed., *From Frege to Gödel*, Cambridge: MA: Harvard University Press 1967, pp. 127–128: “It is all the more serious since, with the loss of my Rule V, not only the foundations of my arithmetic, but also the sole possible foundations of arithmetic, seem to vanish.”

⁶² Bertrand Russell, *Principles of Mathematics*, London: George Allen & Unwin 1903. Cf. “My own view is that, in the end, Russell discovered two paradoxes, one concerning the naive theory of *extensions* and the other concerning *mathematical sets*. In as much as sets are constructed within and for mathematics, Russell’s *set-theoretic* paradox may be solved by *mere stipulation* – much as is done in ZF or the like.” (J C Beal, “Introduction”, in *Liar and Heaps*, pp. 1–5; p. 1, n. 2. Emphasis added in the fourth instance).

Judging by his method of repair, the victim of the paradox of sets was not *the very idea* of sets, but rather a certain theory of sets. The method of repair produced a new theory, this time (as far as we can tell) with the inconsistency left out.

Judging by their method of repair, both Tarski and Russell greatly over-stated the damage done by the respective paradoxes. Each in the end settled for repairs of a type that are wholly compatible with a lesser diagnosis. Interesting as this may be for the history of mathematics and semantics in the twentieth century, the point that requires emphasis here is how this bears on dialetheism. The answer is twofold. (1) At the level of their *stated* diagnoses, dialetheism has a decided edge in cost-benefit terms. (2) But when the diagnoses are read off from the actual modes of their repair, dialetheists lose that cost-benefit advantage. But they don't lose everything. What is still left open, in principle, is a more-or-less even playing field in which the contest is between the respective costs of expunging the contradictions or leaving them be. Of course, *historically*, the playing field is not even, as we have seen. The dominant approaches to sets and semantics alike favour the first of these options to the point of deeply dug-in orthodoxies. But even here the dialetheist can plead the influence of externalities, and can work towards a more receptive future.

13. Trivialism

The Curry Paradox purports to demonstrate that every proposition is true. If that were so, the negation of every proposition would be true, hence every proposition would be both true and untrue. Why do dialetheists resist trivialism so? One possibility is that dialetheists think that trivialism miserably fails the Hippocratean test that their own dialetheism passes. They may think that having everything as true and untrue is a good deal worse than having just a few things as true and untrue. Perhaps this is so, but even if it is, we should certainly want to know in what this worseness actually consists. It is an important question, to which an answer is suggested by how the dialetheist responds to the consistentist's insistence that dialetheism is worse than it.

Why, then, would it be better to have no true contradictions rather than some (albeit not many)? The consistent's answer is that in making true contradictions available as premisses for subsequent reasoning, we let loose the wherewithal for mischievous inferences. But, against this, we don't in fact produce mischievous inferences as long as we take elementary precautions. One is to be suitably selective in the premissory contracts that are let to true contradictions. The other is to constrain our inference-engines paraconsistently. Seen this way, the truth and falsity of the Liar sentence will be a strictly local matter, and the other branches of learning — string theory, neuropsychology, biochemistry, the lot — can go their merry way, wholly undiscommoded by this isolated classical eruption.

It may appear that no such defence is open to the trivialist; and it is this that the dialetheist might want to seize upon. With trivialism, true contradictions are not isolated classical eruptions. Their embrace extends to all that there is, and more. How, then, can true contradictions be stood down for premissory duty in those contexts in which performing it would indeed lead to mischief? How can string theory, neuropsychology, biochemistry, the lot, go their merry way if their every respective pronouncement is true? Isn't this the worseness that afflicts trivialism and ruins it as a rival of dialetheism? If

trivialism were true, how could there be a rationale for there being disciplines at all? If enquiry is the business of determining which propositions are true rather than not, having a demonstration that every proposition is true and not true dispossesses enquiry of its defining task, hence of its very possibility. Doesn't trivialism *actually* deliver the catastrophe that consistentists incorrectly attribute to dialetheism?

In fact, there is reason to think not. We have had access to this reason for at least as long as our intellectual tradition has acknowledged the appearance-reality distinction. The founding insight of the Western philosophical orientation is that, for large classes of cases, false appearances may be corrected not by the deliverance of oracles but by the operations of the intellect. It was never supposed that it be a condition on such corrections that the operations of the intellect cause false appearances to *disappear*. Knowing that the sun, contrary to appearances, does not rise does not change the fact that the sun does indeed appear to rise. Dialetheists (and consistentists too) have invested heavily in the fact that, to say the least, things don't appear to be trivial. With the possible exception of the things dialetheists are dialethic about (see below), things are *experienced* in ways that comport with *LNC*. This solves the premiss-input problem for all the disciplines that constitute enquiry. It enables us to restrict our premisses to propositions that appear to be true and appear not to be untrue. This anyway is how we would transact the business of string theory, neuropsychology, biochemistry, and all the rest, if trivialism were true. It is the same way in which we transact that business *now*.

I imagine that some dialetheists might derive a measure of comfort from these assurances. Failing that, they will have carelessly thrown in with the consistentist on the matter of the worseness of trivialism. It is a fraught alliance, made so, as I say, by the structural similarities between the Liar proof and the Curry proof. This is sharply discouraging to anyone who holds that the one is a sound proof of something surprising and yet the other is a *reductio* of the worst thing ever. The dialetheist must find a way of wrecking the Curry proof without also wrecking the Liar proof. It is widely supposed that this is the Achilles'Heel of the entire dialethic project. But if the dialetheist is prepared to be moved by trivialist assurances, he can lighten up considerably. He should accept those assurances. It is the Hippocratean thing to do. *Or is it?*

In answer to the objection that enquiry (indeed life itself) would be paralyzed if everything whatever were the case, it is open to the trivialist to offer a reply to this effect. "No, to the extent that omni-truth is not a *liveable* fact, do your science and live your life as if consistentism were true". This is not far off Kant's answer to his own scepticism about the world of things-in-themselves.⁶³ Given that the noumenal world can't be penetrated by beings like us, there is nothing for it but to treat the phenomenal world as if it were the real thing. This sets up the dialetheist for the obvious question. If the trivialist

⁶³ Some readers will think my invocation of the name of Kant is rather dubious. I admit to heterodox views about Kant. Granting that it is things in themselves which are real, Kant has the problem of ducking the consequence that the empirical or phenomenal world is the world of mere appearance. To this end, he contrived an ambiguity in the term "real" which it manifestly fails to have, whether in German or English or anywhere else. The purported contrast is between the transcendentially real and the empirically real. The extension of the former is significantly smaller than the extension of the common sense meaning of "real", whereas the extension of the latter lies much closer to that of the common sense term. So nearly everything that is real to common sense is empirically real in Kant's sense. Empirical reality is by and large just common sense reality. But the fact remains that the empirically real are *not* real. Kant tries to disguise this with a tendentious baptism. (I thank Lawrence Powers for helpful instruction on this point.)

can't *live* his contradictions, if he is free to make his science (and do his shopping) on consistentist principles, why shouldn't the dialetheist do the same? Why should he exert the slightest effort to import his contradictions into the science of sets and language? It is here precisely that the dialetheist's intolerance of omni-truth achieves a perch, or anyhow some motivational force. Prospects for embedding trivialism in science and life are hopeless. There is vastly too much to embed. But *select* contradictions might be another matter. In work already done on paraconsistent mathematics⁶⁴ and semantics,⁶⁵ there is evidence that selective embedment is far from hopeless. Like everyone else, dialetheists want to be realists as much as circumstances permit. They want to be able to regard the contradictions they embed (i.e., decide not to extract) to be objectively true. This calls back to centre-stage the Achilles Heel problem for dialetheism. If the Tarski proof is sound how could the Curry proof not be?

There is a reply to all of this. It is a reply that preserves the spirit of the trivialist's attempt to reassure the dialetheist on the score of Curry's proof. It involves re-applying the distinction between science and life. Science embeds all kinds of realities that can't be lived. No one should jibe at embedding something simply on the grounds that it doesn't *inform one's experience*. As a cheerleader for true contradictions, one might think the dialetheist is required to scientifically embed every contradiction that is true. If this is so, it encumbers him with the necessity to paralyze science and enquiry. It is not so. The dialetheist may freely elect to embed the contradictions he *can* embed, and leave it at that.⁶⁶ And he now has a principled reason to favour those contradictions that come his way one by one, from the conceptual inconsistencies of distinct cases, rather than from a proof that delivers the truth of all there is in one fell-swoop. This provides the dialetheist with a rough kind of operational test: Notwithstanding that the Curry establishes that everything is true, embed only those contradictions which also arise from their own independent proofs.

14. The baddest of the bad

I said above that had Tarski actually heeded his own stated diagnosis and triage of the Liar, he would have concluded the non-existence of any natural language. This, I said, would have given a clear-cut advantage to the dialetheist. Even for those who place true contradictions in the category of the bad, surely a sprinkling of true inconsistencies is less bad than the massive illusion of the reality of human languages. Perhaps this is so. In fact, let us concede it. Doing so positions us oddly. For what we are presently absorbed with is reassuring the dialetheist that the costs of omni-inconsistency are not sufficiently high to *require* him to find a differential appraisal of the Liar and the Curry proofs, according to which the Liar proof is just fine and the Curry a *reductio*. But if the truth and falsity of every proposition whatever is an affordable cost, made so by the proto-

⁶⁴ R.T. Brady, "The Non-Triviality of Dialectical Set Theory", in Priest *et al.*, *Paraconsistent Logic: Essays on the Inconsistent*, pp. 437–471. Chris Mortensen, *Inconsistent Mathematics*.

⁶⁵ Priest, *In Contradiction*, pp. 157–187.

⁶⁶ An alternative is that the selective contradictions that dialetheists think are true might in due course successfully infuse our experience of things like sets and self-referential statements. Robert Meyer has suggested this in the case of sets ("Entailment", *Journal of Philosophy*, 68 (1971), 808–818); and Graham Priest has told us of the difficulties he had in getting himself to believe the Liar contradiction, from which we may infer that he does indeed really *believe* it. (Priest, *In Contradiction*.)

Kantian observation that the Curry's truth is not a *liveable* one, why should we not extend the same rescue to the unimaginability of there being no languages? Here, too, could we not say that whereas the proof of the Liar shows natural languages to be a massive illusion, it is not a liveable illusion? What's sauce for the goose, etc. If trivialism is a cost that dialetheists could seriously consider affording, why is not the non-existence of language a cost that a dialetheist could also consider affording? If, in the first instance, the nod might go to the trivialist, why, in the second, might it not go to the Tarskian? Of course, once again, it all boils down to which is the badder of the bad, the non-existence of human language, or the truth of every proposition whatever?

No doubt a kind of Hobson's Choice for consistentists, it is a fair question to put to the *dialetheist*. Why, we ask, would there be reason for him to defer to trivialism but not to the consequences of Tarski's stated triage of the Liar? What earthly difference, reckoned by costs and benefits, could there be between acquiescing in the truth of every proposition whatever and in the truth that language is impossible? There is an answer to this. The dialetheist's accommodation of trivialism turns on his having systematic and coherent sets of experiences that are uninformed by this radical fact (assuming it to be a fact). But if it were also a fact that human language were impossible, human beings would not be possible, and one could not have experiences that are uninformed by this fact. Another way of saying this is that experiencing the world as a human being requires that one *be* one. (*Mimicking* the experience of the world as a human being is another matter).

15. *Self-refutation*

Needless to say, trivialism is a massively radical theory.⁶⁷ Critics (as opposed to mere detractors) might think that it is too radical for its own good. They might be led to suppose that if every proposition is false, then the Curry Proof itself cannot be sound. And if the Curry Paradox cannot be sound, trivialism collapses for want of an adequate demonstration of it. No. Trivialism is the dialectical version of rope-a-dope. It makes antecedent provision for self-refutation, and disarms it by admitting it. If trivialism is true, the Curry argument is unsound. But if trivialism is true, of course the Curry argument is sound. If trivialism is true, trivialism collapses for want of an adequate demonstration of it. If trivialism is true, it enjoys a fully adequate demonstration of it. In the pugilist's conception, rope-a-dope is a means of exhausting one's opponent by inviting barrages that are not defended against. Inside a boxing ring, the unanswered punishment is, to a degree, dissipated on the ropes that the fighter under attack lies back on. It is all a matter of physics. The abuse flows through to the ropes. In the present case, trivialism is the ropes. Every telling blow against the trivialist is nullified by having been antecedently verified by his trivialism. Every attack on it is a confirmation of it. This is not physics, of course. It is dialectics.

This would be the place to harken back, one last time, to the dialetheist's rejoinder to the regulative principle defence of *LNC*. It urges the indispensability of *LNC* as a regulative principle of thought and action. "So it is", was the dialetheist's response, "but in saying so there is nothing that requires *LNC* to be true (only)". As we now see, any

⁶⁷ In Iris Murdoch's novel, *The Book and the Brotherhood*, someone remarks, "Plato did a good job when he threw out the proSocrates" "Yes", was the reply, "But they're back."

dialetheist drawn to this answer is now met with large numbers of chickens come home to roost. If the truth (only) of *LNC* is not a condition of its *regulative* success, it is a matter of indifference as to how *many* contradictions chance to be true.

Trivialism is the least loved of the requisite alternatives. Paying it any degree of significant attention is understandable occasion for impatience, perhaps even for outrage. Just so. Such is a natural concomitant of a cherished presumption under modes of attack for which the wherewithal of effective defence seems hard to come by.⁶⁸ What we learn from this, more valuably than whether the *content* of trivialism is true, is that the blueprint for trivialism is contained in the fundamental epistemological orientation of the philosophical tradition inherited from Attica. It is a hard lesson. Not only does it press us

⁶⁸ It bears on this point that *LNC* has been something like a taboo. Leaving aside various anthropological niceties, taboos are practices that are governed by two prohibitions. One provides that the practice not be *implemented*. The other provides that the practice not be *discussed*; in particular, that it not be questioned or submitted to challenge. This endows taboos with a highly significant dialectical limitation. If taboos are matters against which no challenge is ever heard, they offer no occasion for the *answering* of challenges. The absence of a need to defend the prohibition of a practice generates an inability to defend it if — contrary to what has been the case until now — someone were to put the matter under challenge. Let *P* be a proposition that historically has prohibited the performance of some action protected by a taboo. Let that proposition now be challenged. Then, for anyone who *hears the case against P*, the taboo that has given to *P* its protective coverage now lapses. A decision to hear a case that one is unprepared to answer is like installing software that is congenial to a virus. When the virus attacks *P*, two facts come into a significant concurrence.

First fact. Mere dismissals of the challenge are no longer tolerable.

Second fact. Since the now-challenged proposition has never required the answering of a contrary case, the requisite dialectical resources for meeting it do not now exist.

We have it from the first fact that *P* now requires a proof — or anyhow a defence against the challenger's attack. We have it from the second fact that the required proof cannot be produced. Dialectical incapacity precludes the proof that is now required.

If we think of *LNC* as linked to the taboo prohibiting the characterization of contradictions as true, then *LNC* attracts the same dialectical impotence which the taboo against, say, same-sex marriage has recently occasioned in places like San Francisco and Vancouver. The taboo-like character of *LNC* puts the consistentist in a decidedly odd position. Protected from challenges for most of its long history, *LNC* now *is* challenged (something that didn't happen in 1902), if only by the fact that dialetheism has become a published research programme. Like any lapsed taboo, *LNC* will have attracted the dialectical incapacity that attends such reversals. Hence the shift from dogmatism to economics. For a more detailed consideration of the dialectical structure of historically privileged principles when they come under attack, see John Woods, "Slippery Slopes and Collapsing Taboos", *Argumentation*, 4 (2000), 107–134, and John Woods, "Privatized Death: Metaphysical Discouragements of Ethical Thinking", *Midwest Studies in Philosophy*, XXIV (2000), 199–127.

⁶⁶ The Solver's Paradox presents a problem that resembles what is called the "revenge" problem in the antinomy literature. It was first discussed in connection with Kripke's theory of truth (*op. cit.*). The revenge problem seizes on the claim of any theory that finds the Liar sentence to be defective. Supposing that the "defectiveness predicate" could be expressed in the language of the theory, could the paradox re-appear? The usual treatments of revenge disclose a problematic structure somewhat different from that exposed by the Solving Paradox. For recent discussions of the former, see Hartry Field, "Semantic Paradoxes and Vagueness Paradoxes", in Beal, *Liars and Heaps*, pp. 270, 273ff, 297–307, and Stephen Yablo, "New Grounds for Naïve Truth Theory" p. 328f.

to reflect on the outer limits of case-making and case-answering, it gives us dramatic occasion to examine the bona fides of that inheritance.

16. *The Solving Paradox*

Let us say that a paradox *has a solution* if and only if there is adequate reason to judge it either invalid or a valid *reductio*. Let us say that a paradox has a *standard solution* if and only if there is adequate reason to judge it a valid *reductio*. Suppose that a paradox has a standard solution. Then we may say that there exists for it a *solving-predicate* *S*. For semantic paradoxes such as the Liar, the leading interpretations of *S* include, “does not express a proposition”, “is not a statement”, “is meaningless”, “does not have a truth value”, and “is not true at any level [of the requisite hierarchy]”. Accordingly, with respect to the Liar construction.

(1) (1) is not true

the purported solution is

(2) (1) is *S*

Consider now a claim in the form

(3) (3) is *S*

This gives rise to the *Solving Paradox*, as follows:⁶⁹

Granted that *S* is a solving-predicate for (1), by parity of reasoning there (2) itself also satisfies such a predicate. For how can it be, in relation to

(1) (1) is not true

that (1) is not a statement (or is meaningless, etc.) if, in relation to

(4) (4) is not a statement (is meaningless, etc.)

does express a statement (or is meaningful, etc.)? Accordingly, we may say that if (1) has a standard solution, so too does (3). So if we retain the consistentist assumption that (1) does indeed have a standard solution, we must allow that the same does in fact hold of (3). Consider, for generality,

(5) (5) is *S*

Since (3) has a standard solution,

i) (5) is *S*

ii) If (5) is *S*, then (5) is true

iii) If (5) is true, (5) is not-*S*.

This same reasoning also applies to the Curry Paradox. With regard to the Curry sentence

(6) If (6) then *A*

The received opinion is that it too has a standard solution. So (6) satisfies some or other solving-predicate, according to which

(7) (6) is *S*

But if (6) is *S*, (6) is true, and if (6) is true (6) is not-*S*.⁷⁰

⁷⁰ Stephen Yablo proposes a semantic paradox that turns on factors other than self-reference. The non-referentiality claim stands or falls on whether a sentence in the form “All of the following sentences are untrue” is non-self-referential. See Yablo, “Paradox without Self-Reference”, *Analysis*, 53 (1993), 251–252) Michael Clark observes (*Paradoxes from a to z*, p. 216) that, even so, Yablo’s Paradox might respond

The Solving Paradox is dire news for consistentists. Short of finding a solving-predicate that avoids the fate of the dominant interpretations of *S*, they must try for a non-standard solution of the Liar and the Curry. That is to say, they must try to show that the respective proofs are *invalid*.⁷¹

If there are solving-predicates for the Liar and the Curry, then these proofs are not sound proofs of something surprising. They are not sound proofs of anything. The Solving Paradox shows that, by classical lights, the Liar and the Curry don't admit of standard solutions. This is music to the ears of the dialetheist, provided that he can tell a convincing story as to why his own dialetheism should show its hospitality to the classical *reductio* that the Solving Paradox supplies. It is widely conceded in the dialethic literature that dialetheists are not *in principle* barred from using *reductio* arguments to their own advantage. When they are and when they aren't is a matter of some complexity (and contention). But I propose to give them the nod in the present case, and to allow them the inference that the inconsistency generated by the Solving Paradox suffices to deny to the Liar a standard solution.

The dialetheist should consider refusing the offer. The inconsistency generated by the Solving Paradox also denies to the Curry a standard solution. If the failure of the Liar to have a standard solution is reason to accept dialetheism, then the failure of the Curry to have a standard solution is equal reason to accept trivialism.

This leaves the dialetheist in a bind. Like the consistentist, he must find a non-standard solving-predicate that applies to the Curry but not to the Liar, or like the consistentist he must try to invalidate the Curry (but without invalidating the Liar) or he must acquiesce to trivialism.

This is quite a lot for the dialetheist to be thinking about. It might ease his burdens if he were able to take to heart our "Kantian" suggestions that trivialism might not be as bad as it seems. Short of that, he might consider not being a dialetheist.

Ah, yes. But what about *negation*?

17. Negation

In a scornful passage of considerable celebrity, Quine writes,

My view . . . is that neither party knows what he is talking about. They [sic] think they are talking about negation, '~', 'not'; but surely the notation ceased to be recognizable as negation when they took to regarding some conjunctions of the form ' $p \bullet \sim p$ ' as true, and stopped regarding such sentences as implying all others. Here, evidently, is the deviant logician's predicament: when he tries to deny the doctrine he only changes the subject.⁷²

On a tight reading, this is a plea for classical negation as the sole candidate for negation that passes muster. Perhaps a more charitable interpretation would allow the nod to be given (albeit grudgingly and tentatively) to intuitionistic negation, and possibly some

to the view that "none of [these sentences] can be allowed to express a statement. Accordingly self-referentiality is not a condition of a paradox's responsiveness to a solving-predicate.

⁷¹ In my view, this is very much the way to go. Doing so pits us against the positions discussed in this paper, and would take us further than there is space for here. But see Woods and Armour-Garb, *Dialetheism Pro and Con: Could Contradictions Be true?*

⁷² *Philosophy of Logic*, p. 81

varieties of modal negation. But the charitable interpretation is tactically damaging to Quine’s intent. If classical negation is not all there is to negation, on what basis is admittance denied to paraconsistent and dialethic negation? On the other hand, if the strict interpretation of Quine’s remark is persisted with, they are twice-over an embarrassment. For, so interpreted, the claim is question-begging. It is also clearly untrue.

In the earliest systematizations of logic, negation subdivides into the notions of contradictoriness, contrariety and subcontrariety. In book Γ of the *Metaphysics*, Aristotle endorses LNC ($\neg \diamond (A \wedge \neg A)$) and *LEM* ($\Box (A \vee \neg A)$). In *De Interpretatione*, he appears to change his mind about *LEM*, now interpreting it as the claim that *sooner or later* it must be that either *A* obtains or $\neg A$ obtains. In the logic of the syllogism, ‘negation’ operates in such a way as to licence the syllogism

$$\begin{array}{l} \text{All } A \text{ are } B \\ \text{No } B \text{ are } C \\ \hline \text{No } A \text{ are } C \end{array}$$

In *Prior Analytics* 57^b3, Aristotle puts it that contradictories cannot entail one another, and derives from this the further claim that contradictories cannot both imply the same thing.

Aspects of Aristotle’s view of negation are given considerable emphasis by mediaeval logicians such as Boethuis and Abelard. By these lights, the reason that no proposition can entail its own negation is that the negation of a proposition “entirely expels and extinguishes it”.⁷³

This, the *cancellation view* of negation, also provides that nothing can entail a sentence and its contradictory, thus establishing a link with *connexivist logic*.⁷⁴ Even so, cancellation is stronger than *connexivism*. If $\neg A$ extinguishes *A*, then it is hard to see how $A \wedge \neg A$ can express any proposition. But if $A \wedge \neg A$ is thus contentless, it entails nothing.⁷⁵

If we leap ahead to Boole’s *Mathematical Analysis of Logic*, we find that although “If *A* then it is not the case that *A*” is false on both the Aristotelian and cancellation approaches to negation, there are instances in which it does not fail Booleanly. Moreover, Boole’s notion of negation is not explosive, as Priest points out.⁷⁶

What this brief review allows us to see is that will before classical negation appeared on the scene, at least three different, and pairwise incompatible, notions of negation achieved a substantial purchase in the development of logic. This presents classicists of Quinean disposition with the obvious question: “Do you find Aristotelian, cancellationist and Boolean negation to be unrecognizable?” Since the question answers itself (in the negative), we may take it that Quine’s *particular* worry was that in allowing for true sentences of the form $A \wedge \neg A$, i.e., in allowing for *dialethic* negation, negation “ceased to be recognizable”.

⁷³ L.M. De Rijk, editor, *Petrus Abealardus; Dialectica*, Assen: van Gorcum 1970; p. 290.

⁷⁴ Storrs McCall, “Connexive Implication”, *Journal of Symbolic Logic*, 31 (1966), 415–433.

⁷⁵ While influenced by Aristotle, cancellationist negation is not Aristotle’s negation, which allows that contradictories sometimes do (and sometimes don’t) have entailments.

⁷⁶ Graham Priest, “Paraconsistency and Dialetheism”, in Dov M. Gabbay and John Woods, editors, *Handbook of the History of Logic*, Amsterdam: Elsevier, to appear; § 7.

This leaves us with two questions. One is whether it is true. The other is, if true, whether it matters. Since this is not the place to produce the full case for dialetheism (or to answer it), a certain programmatic briskness might not go amiss. *A propos* the first question, it is wholly fitting to press the Quinean to tell us what it is about dialetheism that causes negation to disappear? Could it be that there are dialethic systems in which negation takes any sentence whose value is the composite (T, F) into a sentence with the same value? If so, this is not a distinctive feature of dialethic logics; it also plays prominently in certain systems of many-valued logics.

Could it be, then, that the English word ‘not’ when it functions as negation has only so many different senses, and the dialethic sense is not on the list? If so, the intuitionist ‘not’ also makes negation unrecognizable since “has no proof” is manifestly not a sense of ‘not’ in English.

The more important question is the second. Suppose that dialetheism made negation unrecognizable. Why would this matter? It is widely supposed that one of logic’s triumphs has been to have shown how its target properties, such as entailment, validity, logical truth and consistency, are analyzable in depth (and with considerable accuracy) in a language whose only interpreted symbols are the logical particles \neg , \vee , and \therefore (or some other equivalent assortment). It is easy to see that for this to be true, the interpretations given to \neg , \vee , \therefore must be such that the ensuing accounts of entailment, validity and the like come out (approximately) right. It might also be supposed that in order to get entailment (*et al.*) right one must get \neg (*et al.*) right. This is true. But getting \neg right means no more than assigning it an interpretation which, together with the readings of the other particles, allows us to get *entailment* (*et al.*) right. That is to say, getting \neg right is getting \neg to be right for entailment. It cannot be part of getting \neg right that its interpretation captures an antecedently existing sense of the English particle ‘not’, since (again) were it so intuitionist negation would then be expelled.

What, then, would a logic look like which, though it produced a quite decent analysis of entailment, did so on the basis of an interpretation of ‘not’ that made negation unrecognizable?

Consider a case. Consider one of the standard semantics for *first degree entailment*. (*FDE*).⁷⁷ *FDE* is a propositional logic for which there exists a “Routley interpretation”. We put it that every world W has a unique *star world* w^* . Then a Routley interpretation is a structure $\langle W, *, v \rangle$ in which W is a set of worlds, $*$ is a function from worlds to worlds satisfying the condition that $**w = w$, and v is a truth value assignment for each atom of *FDE* at each world. T and F are the sole truth values. We have it that

$$v_w(A \wedge B) = T \text{ if } v_w(A) = T \text{ and } v_w(B) = T, \text{ and otherwise is F.}$$

$$v_w(A \vee B) = T \text{ if } v_w(A) = T \text{ or } v_w(B) = T, \text{ and otherwise is F.}$$

$$v_w(\neg A) = T \text{ if } v_{w^*}(A) = F, \text{ and otherwise is F.}$$

Remarks

Exactly one of A , $\neg A$ is T in w or its star world w^*

If $w = w^*$, the semantics of *FDE* is classical.

Validity and logical truth are defined in the usual way.

⁷⁷ Richard Routley and Valerie Routley, “Negation and Contradiction”, *Rivista Columbiana Matemática*, 19 (1985), 201–231.

FDE is sound and complete with respect to this semantics.

FDE also has a sound and complete relational and many valued semantics.

The three semantics are equivalent. They give the same account of entailment. In the relational semantics, \neg is defined as usual (except that it is relational rather than functional). However, since $*$ lacks an intuitive semantics, \neg in the Routley interpretation inherits this problem and it may be said that makes negation unrecognizable. But, again, the $*$ -semantics delivers the goods for entailment every bit as well as the relational semantics. Thus a system in which negation is unrecognizable is as good an account of entailment as an equivalent system in which negation is not unrecognizable.

Supposing, then, that *dialetheism* also makes negation unrecognizable, why would this matter?

18. Concluding remarks

At various junctures of the foregoing discussion, I have been at pains to emphasize that this is an essay in dialectics, a branch of the theory of argument that deals with resolution strategies for intractable disagreement. It lies in the non-white-coat nature of the disagreements under review that they might well resist satisfactory resolution. There are no guarantees. One of the things that makes this so is that for nearly everyone there is some or other matter he regards as non-negotiable. Notwithstanding recurrent perturbations from presocratics, idealists and mystics, *LNC* must surely top the league-table as human history's least negotiable pronouncement.

Dialetheists want to bring supporters of *LNC* to the table. They want to poke the bruise left by Lewisian dismissiveness. "If you have nothing to say for *LNC*", they say, "what justifies your scorn of the possibilities we raise against it?" It is a seductive remark, as witness Aristotle's striking collapse in *Metaphysics*, book I, from his own steadfastness about the non-negotiability of this necessary, prior and most certain of first principles.

Unlike Aristotle, consistentists are likely to stay away from the dialetheist's table in droves. It doesn't matter. The dialetheist has already shown his hand. Since *LNC* admits of no defence sufficiently conclusive to shut down the very idea of a dialetheic challenge, the dialetheist will challenge the consistentist on economic grounds, and will do so with the full backing of philosophical pragmatism about logic. This too is enormously seductive, for it would appear that consistentists would have no trouble in winning a cost-benefit contest at every turn. We have already seen that, as regards the centre-pieces of dialetheic optimism, consistentist responses to the Russell and the Liar have long-since established themselves as *de facto* winners, never mind their own occasional pairwise incompatibility. As we have noted, winners (like sitting congressmen) have a clear economic advantage. It is costly to bring them down. This is little more than logicians like Quine have recognized for years. Consistentism is best on economic grounds. The grounds might shift, but before *LNC* is compromised, "the returns have better be good".⁷⁸

The fact remains that the extent to which dialetheic blandishments move consistentists from dogmatic to economic confidence in *LNC*, the dialetheist has achieved

⁷⁸ Quine, *Philosophy of Logic*, p. 86.

something of importance. He has got *LNC* on the table. It is no mean feat. Once there, *LNC* does not fare as well in the matter of *direct* justification as one might have expected. Dialetheists would go further. Considered as a universal principle of logic, *LNC* doesn't fare well at all. The extent to which this is so is a tactical victory for the dialetheist. The defences of *LNC* now shift from the direct to the indirect, to questions of what are the better diagnoses and triages of paradoxical proofs. The advantage that redounds to the dialetheist is that exceptions to *LNC* are recognized as possible in principle on economic grounds, never mind the low likelihood of their realization in practice. This is a significant move from kitty-bar-the-door dogmatism. And it exposes the dialetheist to the further advantage that costs and benefits are intrinsically tied to shifting circumstances. However unlikely it may be that mathematics and semantics will in future evolve dialetheically (even aside from the Solving Paradox), it is not foreclosed that this should happen. The proof of the pudding is in the eating. For some time now, such evolutions have been considered possible by consistentists only as expressions of system-relative manifestations of technical virtuosity. But with *LNC* now on the table, such evolutions must be countenanced in less relativistic ways.

We have in this an important dialectical lesson. It is that the system-relative hyphenation of heterodoxies is imperfect protection of any orthodoxy. In retrospect, one can now see that the jig was up for dogmatism about *LNC* once the mainstream allowed dialetheic waters to mingle there. There is a lesson in this. If we don't want the orthodoxy disturbed, then don't give the opposition a hearing; in particular, don't give it a patronizing hearing.⁷⁹

⁷⁹ My research on the structure of inconsistency is supported by the Engineering and Physical Sciences Research Council of the United Kingdom, the Social Sciences and Humanities Research Council of Canada, the Faculty of Arts, University of British Columbia, and the Faculty of Arts and Science, University of Lethbridge. I am most grateful for this assistance. For helpful, need I say not always sympathetic, criticism and advice, I thank Jonathan Adler, Bradley Armour-Garb, J C Beal, Joachim Bromand, Bryson Brown, Charles Daniels, Dov Gabbay, Tony Hunter, Andrew Irvine, Ray Jennings, Erik Krabbe, Lorenzo Peña, Larry Powers, Graham Priest, Greg Restall, David Ripley, Peter Schotch, Harvey Siegel, Howard Sobel and Roy Sorenson. I also wish to record my gratitude to this journal's anonymous referees for valuable suggestions and to Carol Woods for technical support.

Appendix

A question we have left “on the table” is whether the respective structures of the Liar proof and the Curry proof supports the dialetheist’s markedly different diagnoses of them. For readers who may wish to ponder this matter, the two proofs are informally set out here.

The Liar

1. The following item, which we number as ‘(1)’ for expository convenience, is a statement (of English).
 - (1) (1) is untrue.
2. If true, then what (1) says is so, hence (1) is not true by the content of (1)
3. If untrue, then (1) is as (1) says it is so; hence (1) is true. by the content of (1)
4. Therefore, from (3) and (4), we have it that (1) is true if and only if (1) is untrue by the definition of biconditionality

The Curry

1. The following item, which we number as ‘(2)’ for expository convenience, is a statement (of English).
 - (2) If (2) is true, then A (for arbitrary A)
- Suppose
3. (2) is true
 4. “If (2) is true then A” is true by substitution of 2 in (2)
 5. “If (2) is true then A” is true iff if (2) is true then A. by the principle that X is true iff X
 6. If (2) is true then A by modus ponens on (4) and (5)
 7. A by modus ponens on (3) and (6).
- Suppose now
8. (2) is not true
-
9. It is possible both that by the principle that if \neg If X then Y is

- (2) is true and A is false.
10. If it is possible that (2) is true and A is false it is possible that (2) is true
11. If (2) is true then A
12. So (10) is false
13. So, (9) is false
14. So (2) is true
15. Hence A.
- false, then it is possible that both X is true and Y is false
 by the content of (2)
 by the modal principle,
 $\Diamond(A \wedge B) \rightarrow \Diamond A$
- by (3), (7)
- by the principle that
 if $\neg X$ is not true⁵ is false, then X is true
 by the content of (2)